ESC-TR-2004-086

# Correlation of Nonlinear Distortion in Digital Phased Arrays: Measurement and Mitigation

D.J. Rabideau
L.C. Howard

Presented at the 2002 IEEE MTT-S International Microwave Symposium, 4 June 2002

Issued 1 November 2004

**Lincoln Laboratory**
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
*LEXINGTON, MASSACHUSETTS*

Prepared for the Department of the Air Force under Contract F19628-00-C-0002.



101-1037

This report is based on studies performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology. This research effort was sponsored under the auspices of the Lincoln Laboratory Advanced Concepts Program. The Advanced Concepts Program is supported principally by the Department of the Air Force under Contract F19628-00-C-0002.



This technical report has been reviewed and is approved for publication.

**FOR THE COMMANDER**

Gary Tutungian
Administrative Contracting Officer
Plans and Programs Directorate
Contracted Support Management

# Correlation of Nonlinear Distortion in Digital Phased Arrays: Measurement and Mitigation

Daniel J. Rabideau
L. Cole Howard

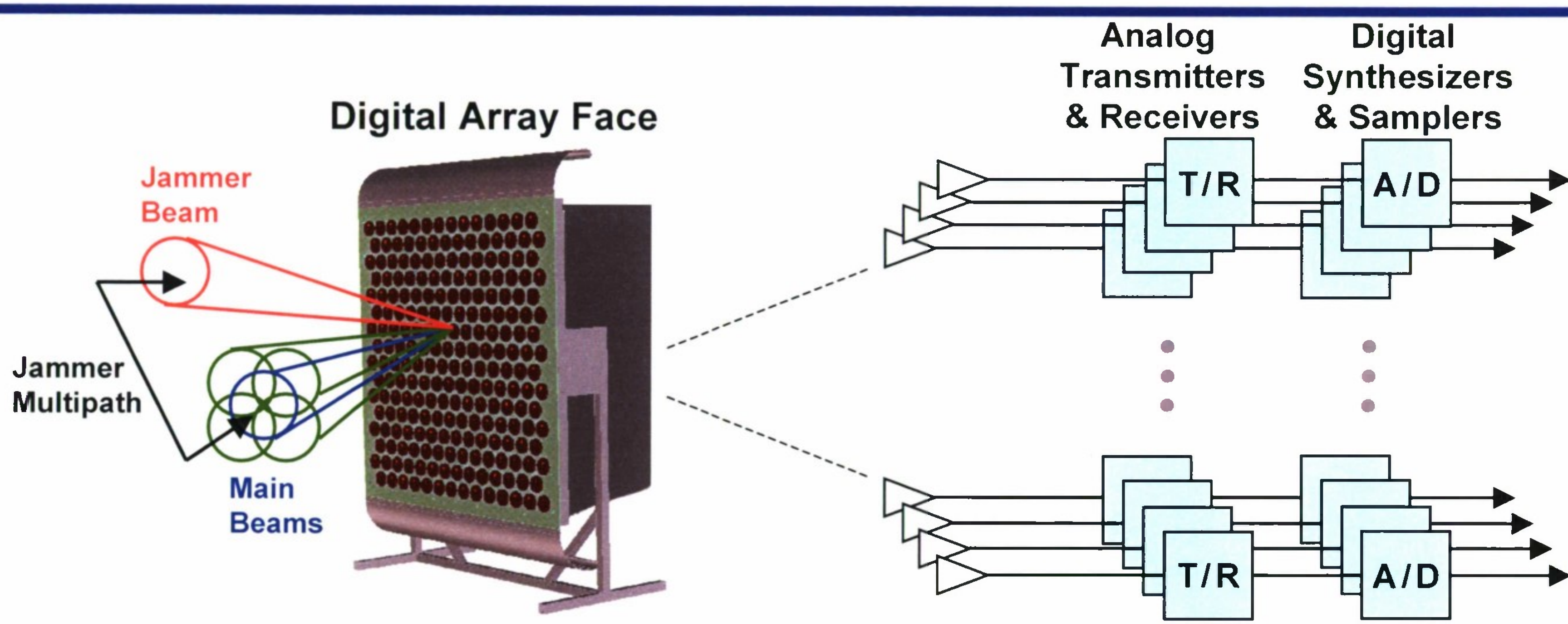
Lincoln Laboratory, Massachusetts Institute of Technology

Presented at the 2002 IEEE MTT-S International Microwave Symposium, 4 June 2002

This work was sponsored under Air Force Contract F19628-00-C-0002.
Opinions, interpretations, conclusions and recommendations are those of the author
and are not necessarily endorsed by the Department of Defense.

50 years
MIT LINCOLN LAB

# Digital Arrays & Their Benefits




- Digital sampling & filtering behind each traditional analog receiver
  - Digital Synthesis (DDS) in front of each traditional analog transmitter
- Enables:
  - Multiple simultaneous beams; Adaptive beams
  - Flexible scan patterns; Faster area search
  - Improved dynamic range & linearity
  - Potential for modular, open-systems design

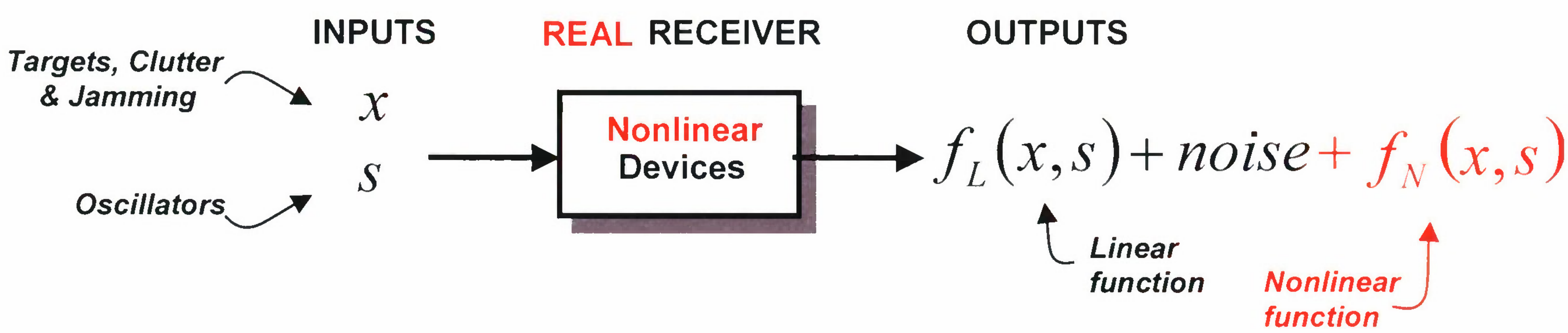
*The subject of this presentation*

# Outline

- Introduction
- Importance of linearity and dynamic range
- Method for improving dynamic range
- Experimental data collection
- Summary and future work

# Linearity and Dynamic Range

- An idealized receiver performs linear operations




- Real receivers attempt to approximate this ...

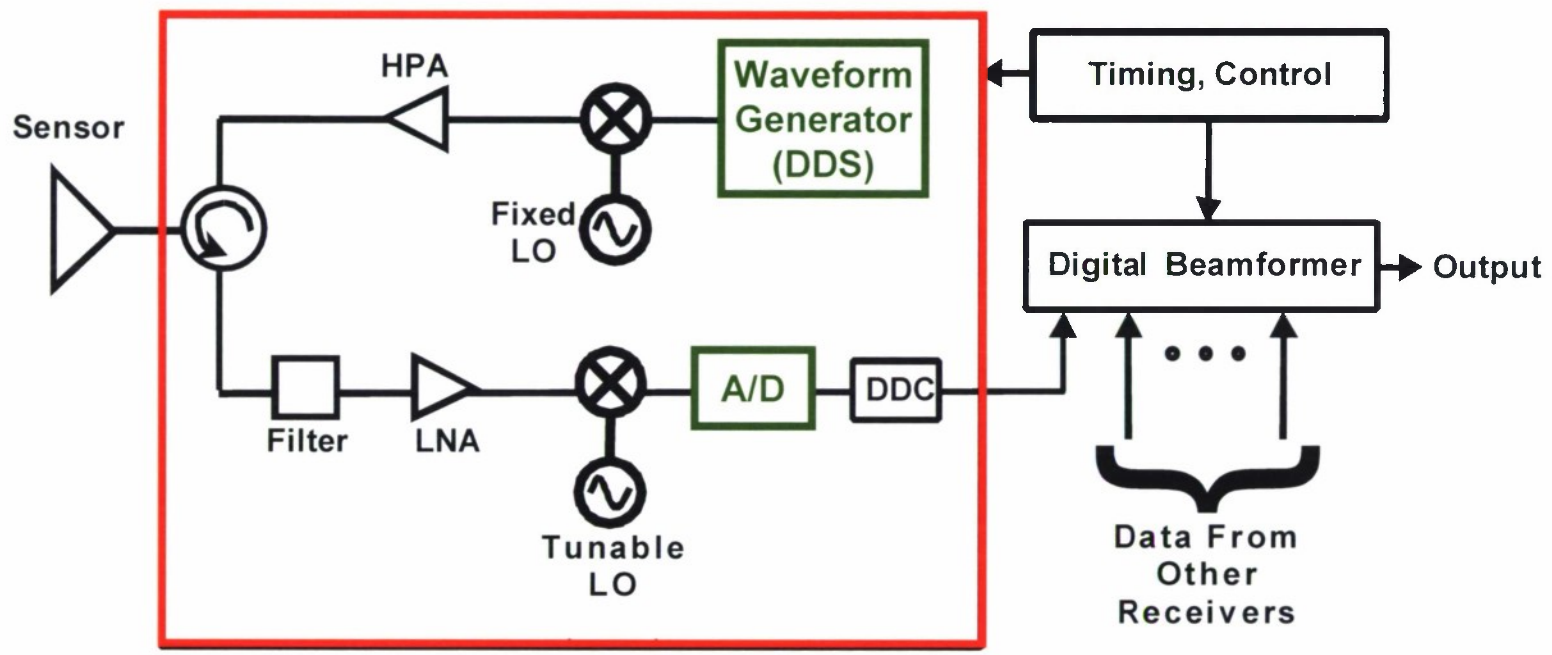
- If nonlinear term exceeds noise, it prevents noise limited detection, i.e., it decreases radar sensitivity

$$\text{Dynamic Range} = \frac{P_{max}}{P_{min}}$$

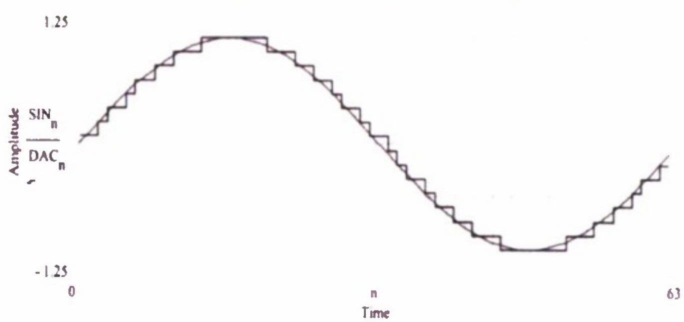
$P_{max}$: Power of strongest signal receivable w/o generating a detectable nonlinearity

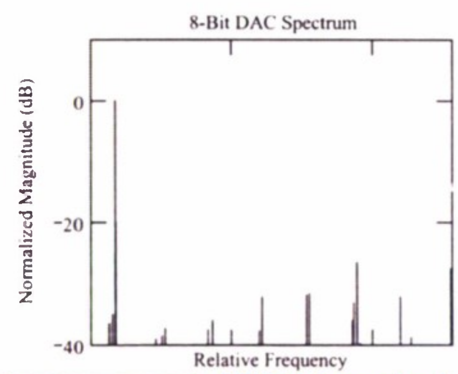
$P_{min}$: Weakest detectable signal

# Some Types of Nonlinearity in a Typical Digital T/R Module

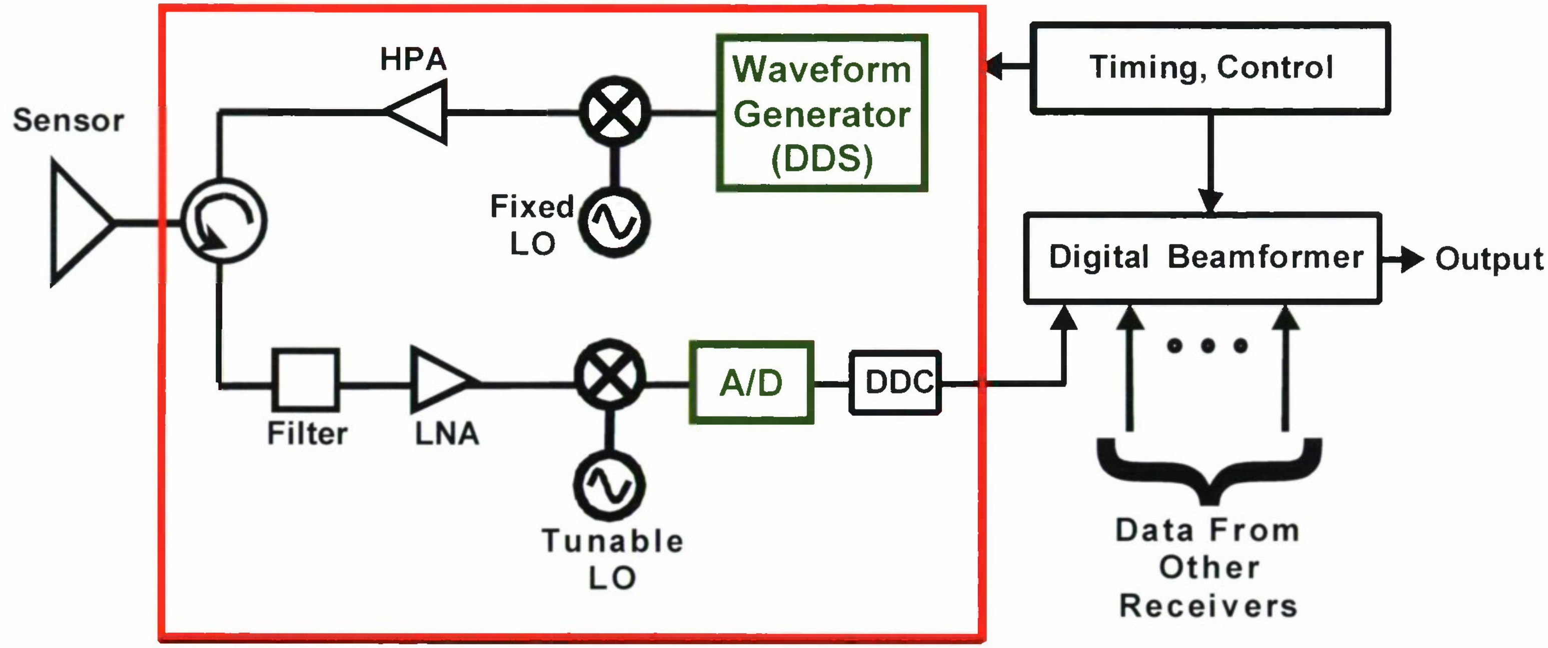



- Quantization (A/Ds, DACs)

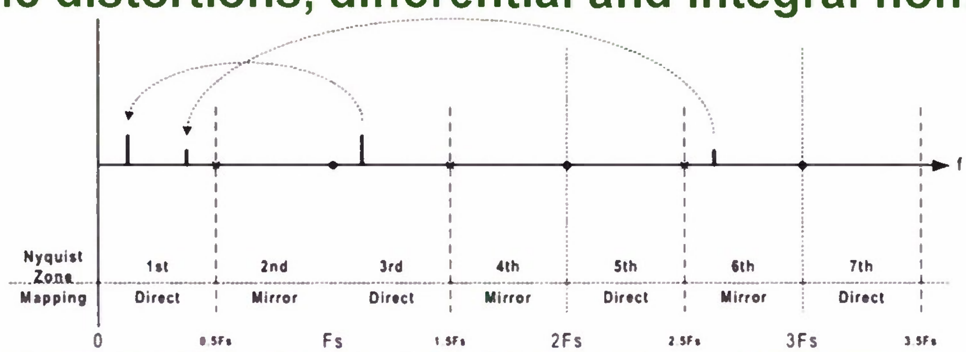


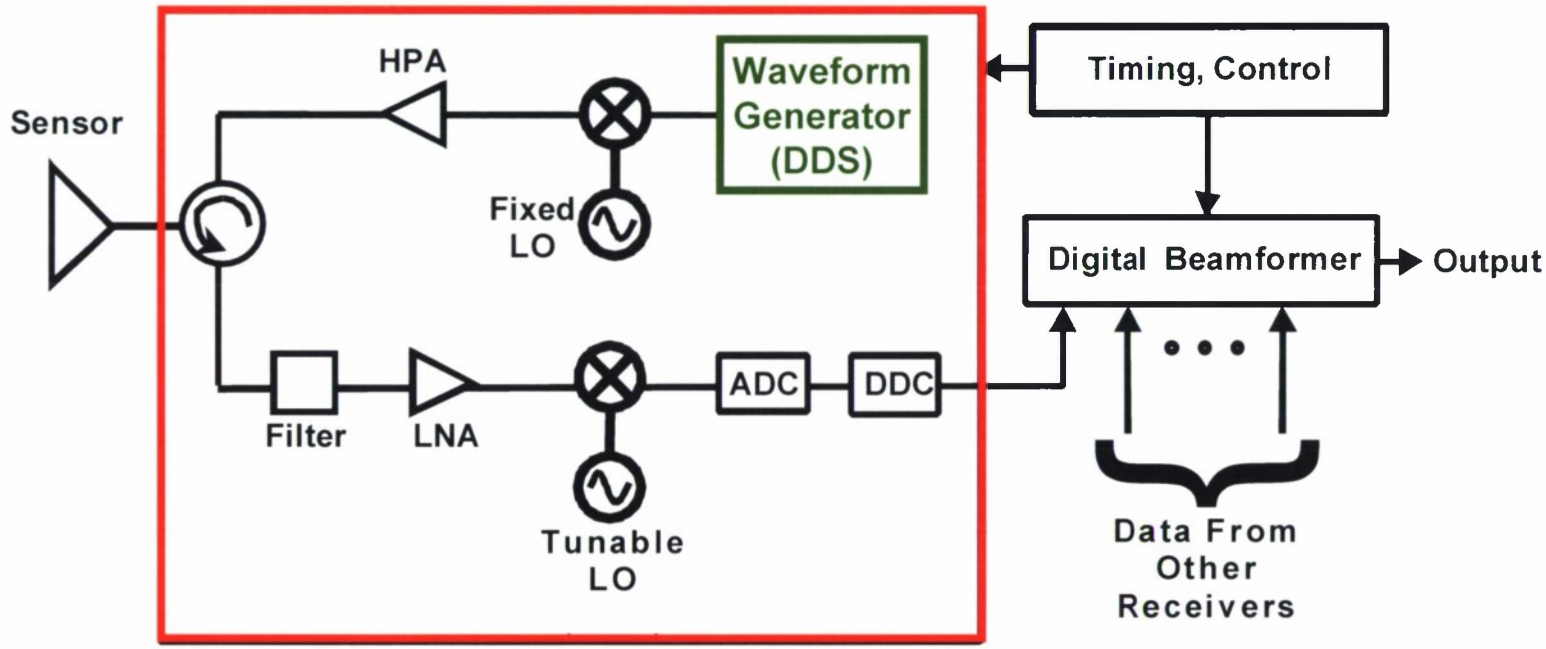
# Some Types of Nonlinearity in a Typical Digital T/R Module



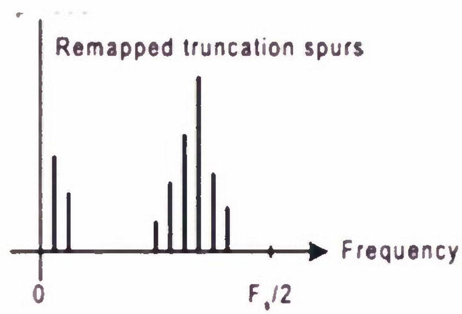

- Harmonic distortions, differential and integral nonlinearities



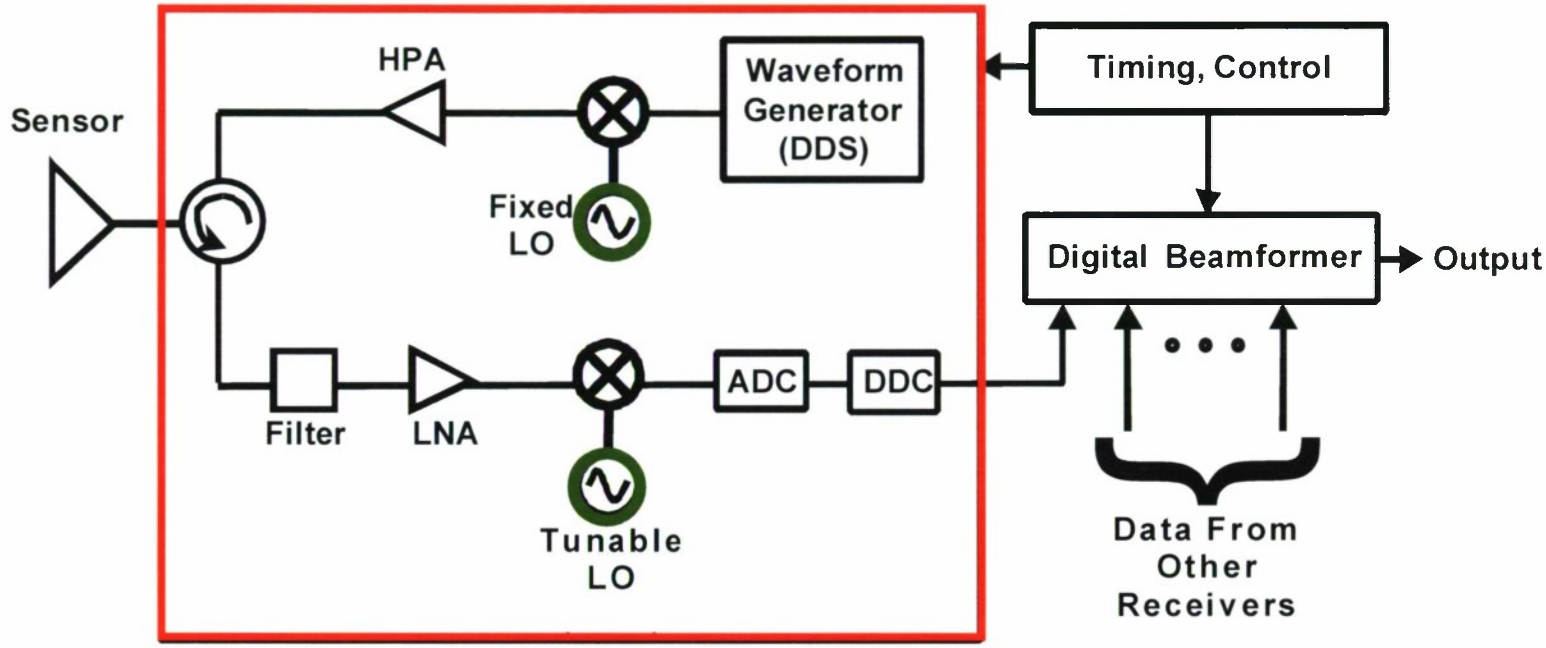
# Some Types of Nonlinearity in a Typical Digital T/R Module



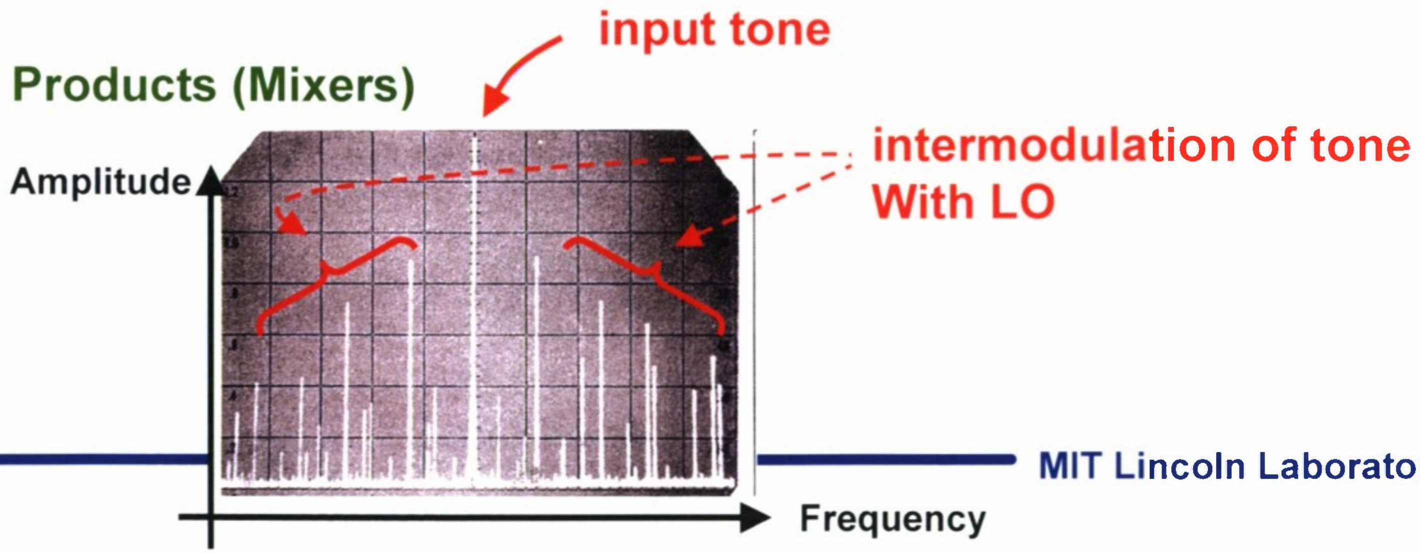

- Phase Truncation (DDSs)



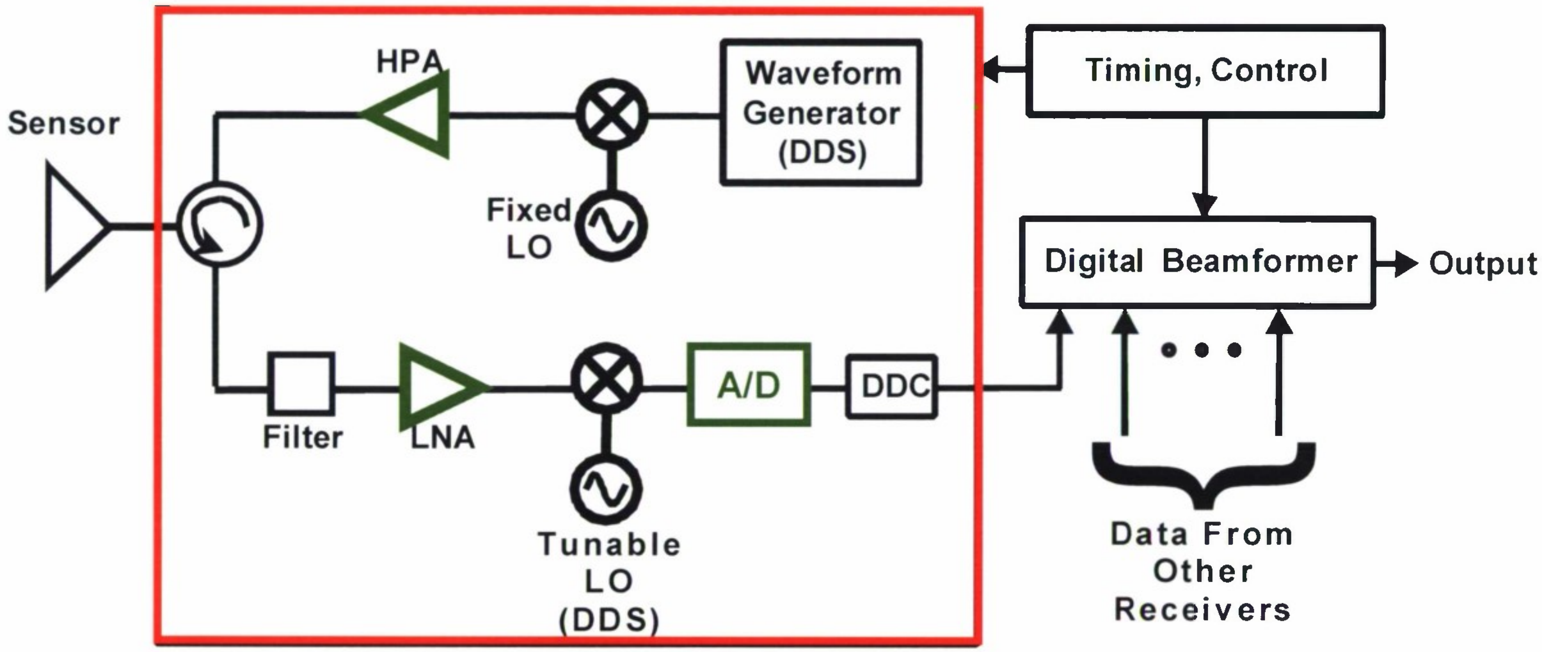
# Some Types of Nonlinearity in a Typical Digital T/R Module



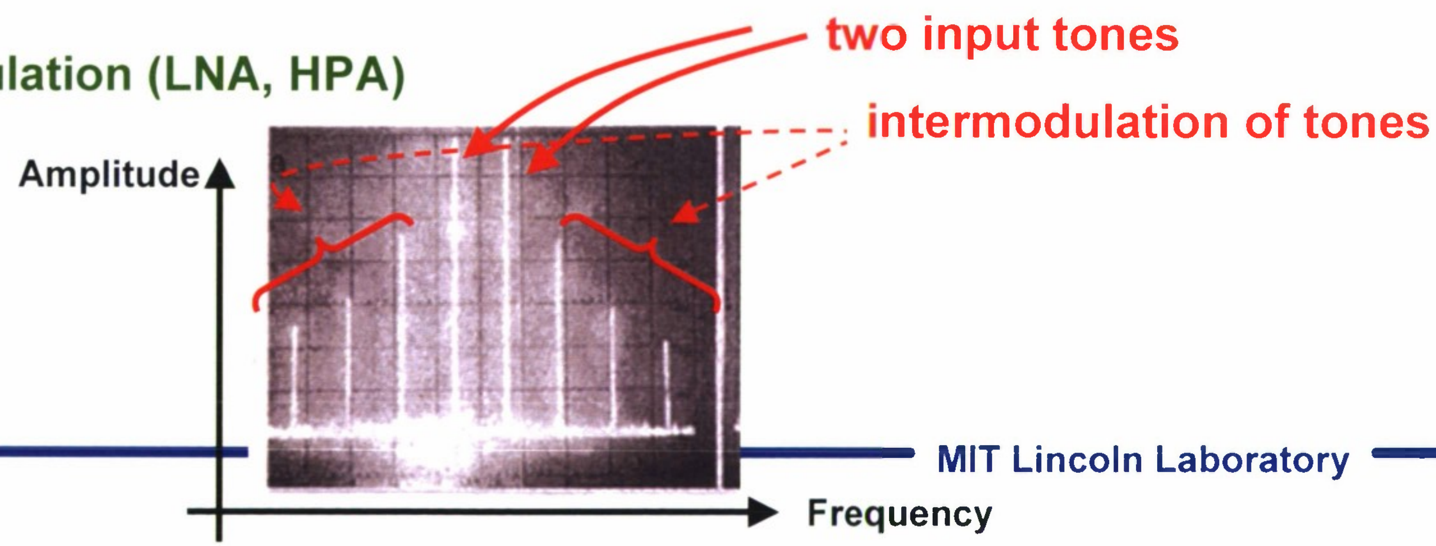

- M x N Products (Mixers)



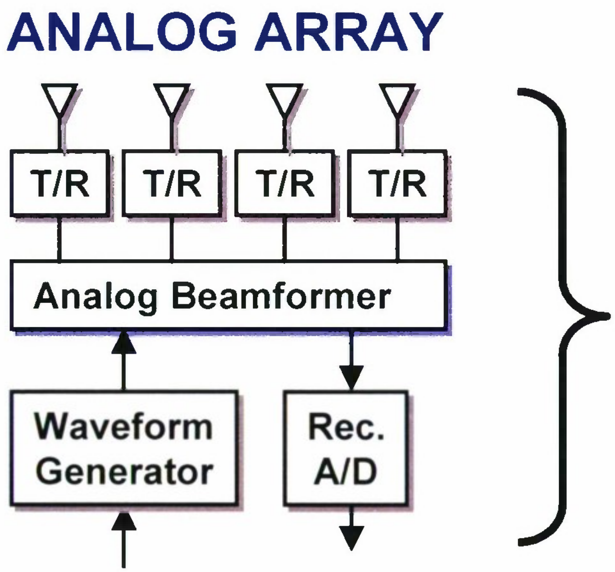
# Some Types of Nonlinearity in a Typical Digital T/R Module




- Intermodulation (LNA, HPA)

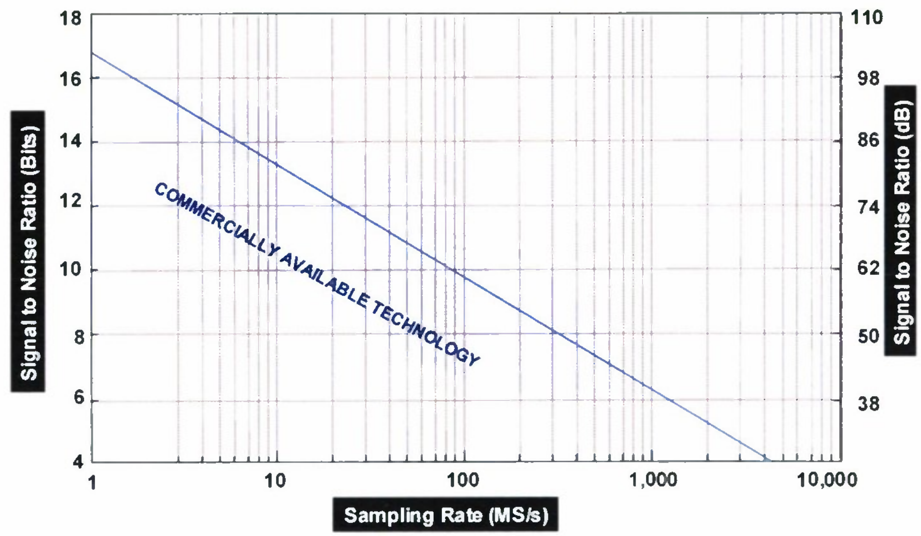


# Linearity Requirements in Analog Arrays

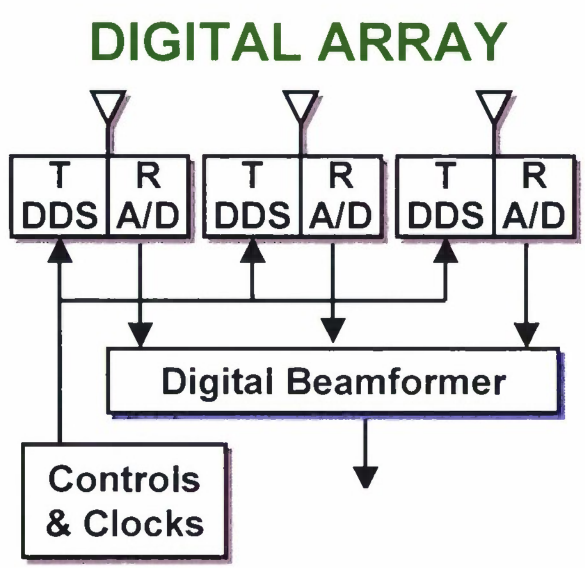



$$\text{Dynamic Range} = \frac{P_{max}}{P_{min}}$$

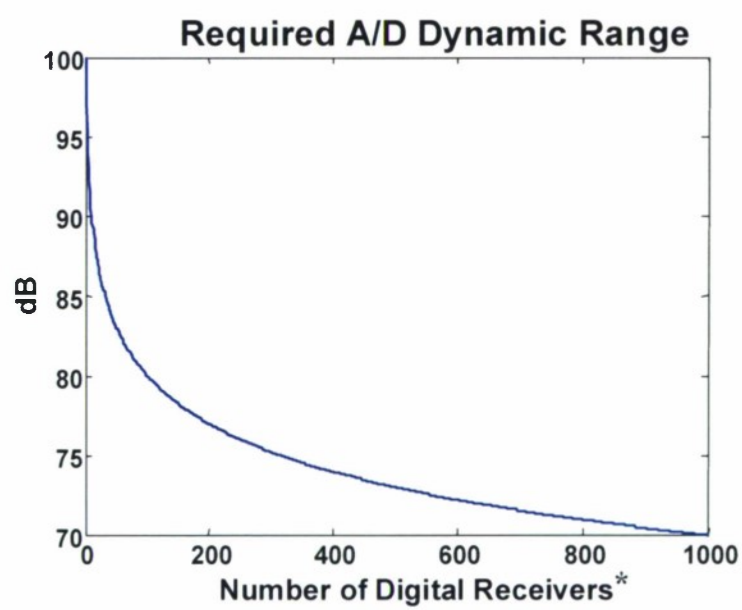
**Due to array gain, the dynamic range requirement is usually greatest at the output of beamformer (i.e. the A/D requires the greatest dyn. range)**



# How Digital Arrays "Theoretically" Improve Dynamic Range




- Digital arrays move the waveform generator and A/D closer to the elements
  - Sampled signals are smaller
  - Requirement on A/D, DDS dynamic range is lessened



# … but there's one BIG problem

- **Even if each receiver's nonlinearities lie below the noise level, the nonlinearities may be correlated spatially (or even coherent).***

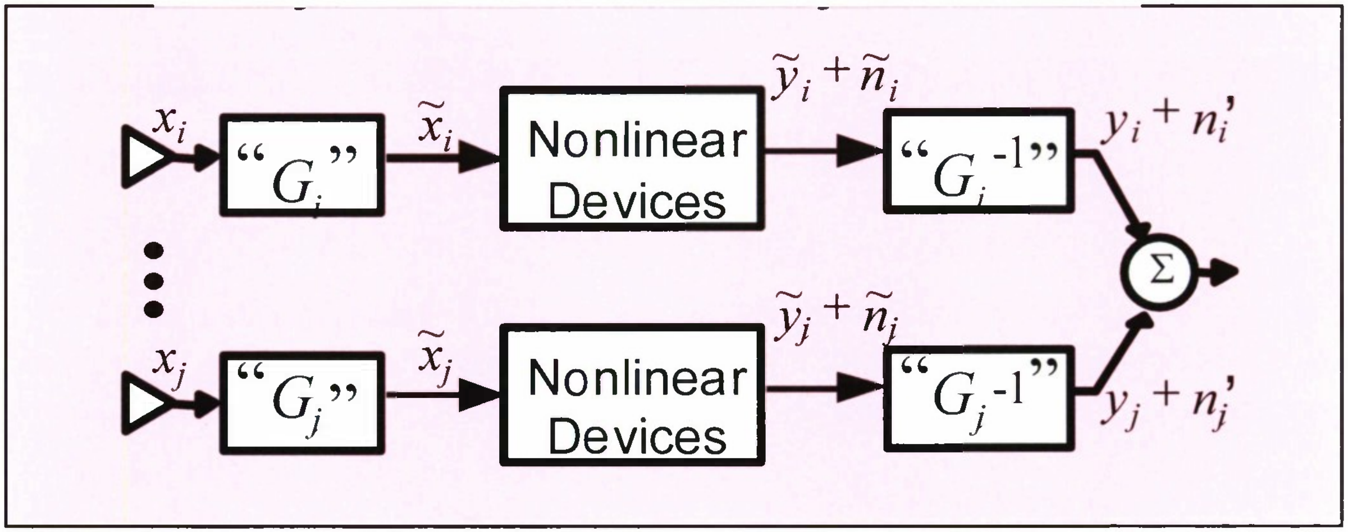
  **If so, the digital beamformer will integrate them (spatially) above the noise, limiting the *system's* overall dynamic range**

** [ We had originally conjected (and proven via mathematic models) that many of these nonlinearities would be correlated. Thanks to the ADC, we've also collected data to show this experimentally … as you shall see. ]*

# Outline

- Introduction
- Importance of linearity and dynamic range
- Method for improving dynamic range
- Experimental data collection
- Summary and future work

# Novel Approach:
## -- Decorrelation of Array Nonlinearities --

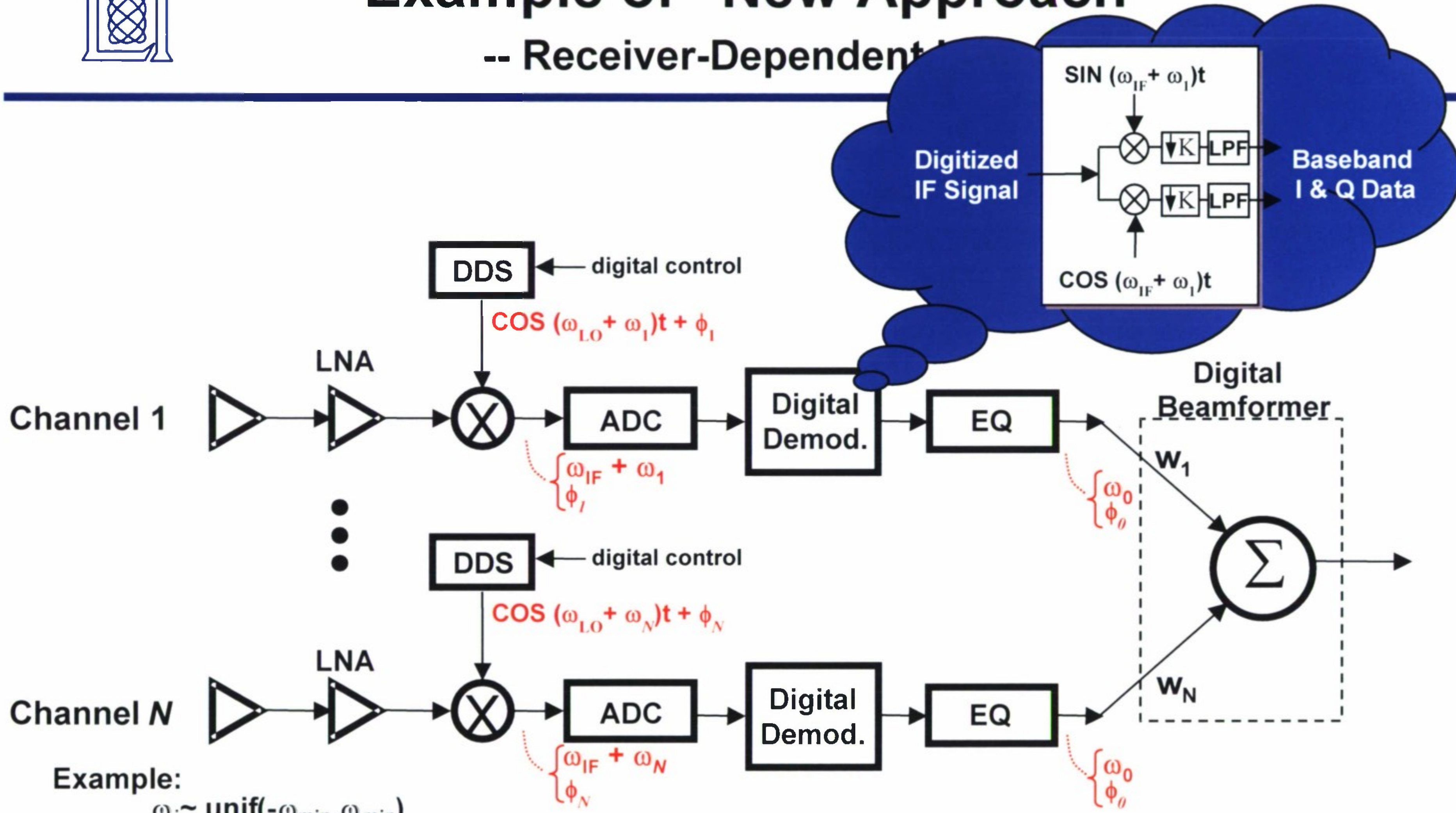



- Input signals are transformed differently at each channel
- "Inverse" transformation later restores the linear component
  - But nonlinear terms still vary from channel to channel, decorrelating them and/or making them sum incoherently.

**NOTE: We do not <u>remove</u> nonlinearities …**
**but we do prevent them from being correlated (i.e., no array gain)**

# Example of "New Approach"

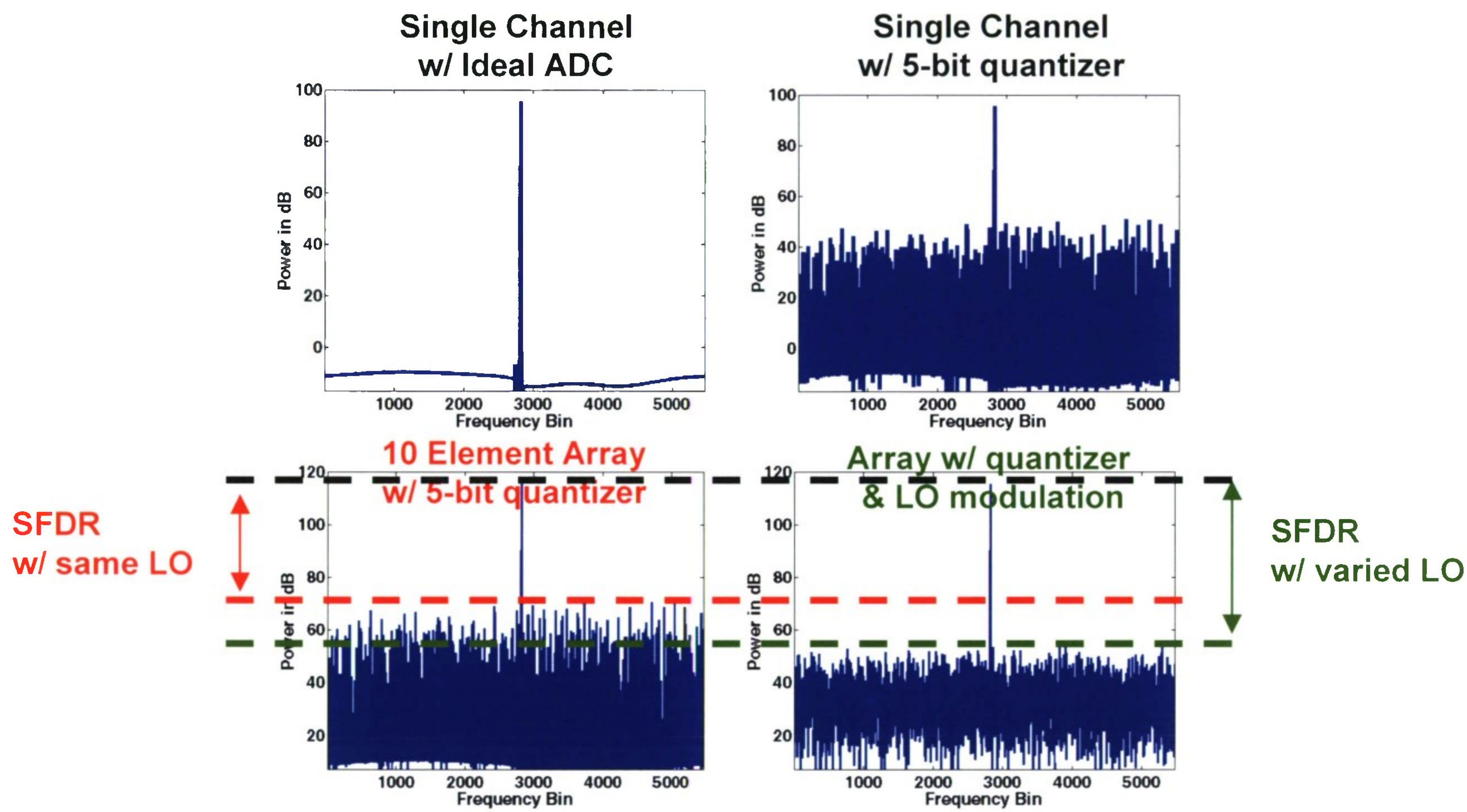
## -- Receiver-Dependent




Example:
$\omega_i \sim$ unif$(-\omega_{min}, \omega_{min})$
$\phi_i = 0$

- Addresses: Harmonically related spurs generated by ADC, DDS and DAC, MxN products in mixer, and LO and ADC phase noise
  - Varied LO causes different sets of frequencies to be excited

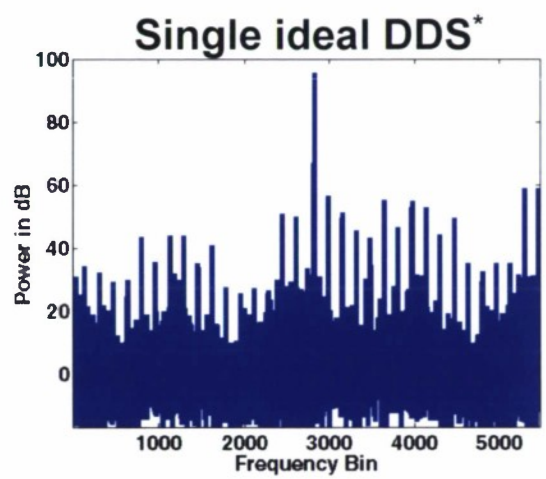
# Predicted A/D Nonlinearities with Receiver-Dependent LO

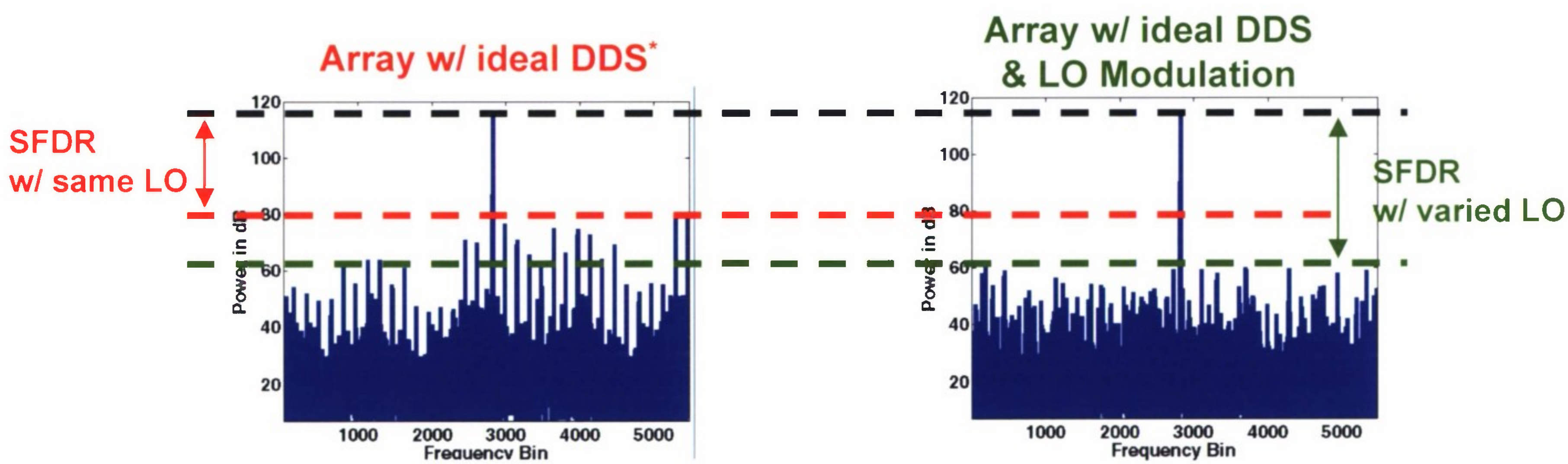



- Randomized LO modulation makes spurs appear uncorrelated, thus avoiding array gain

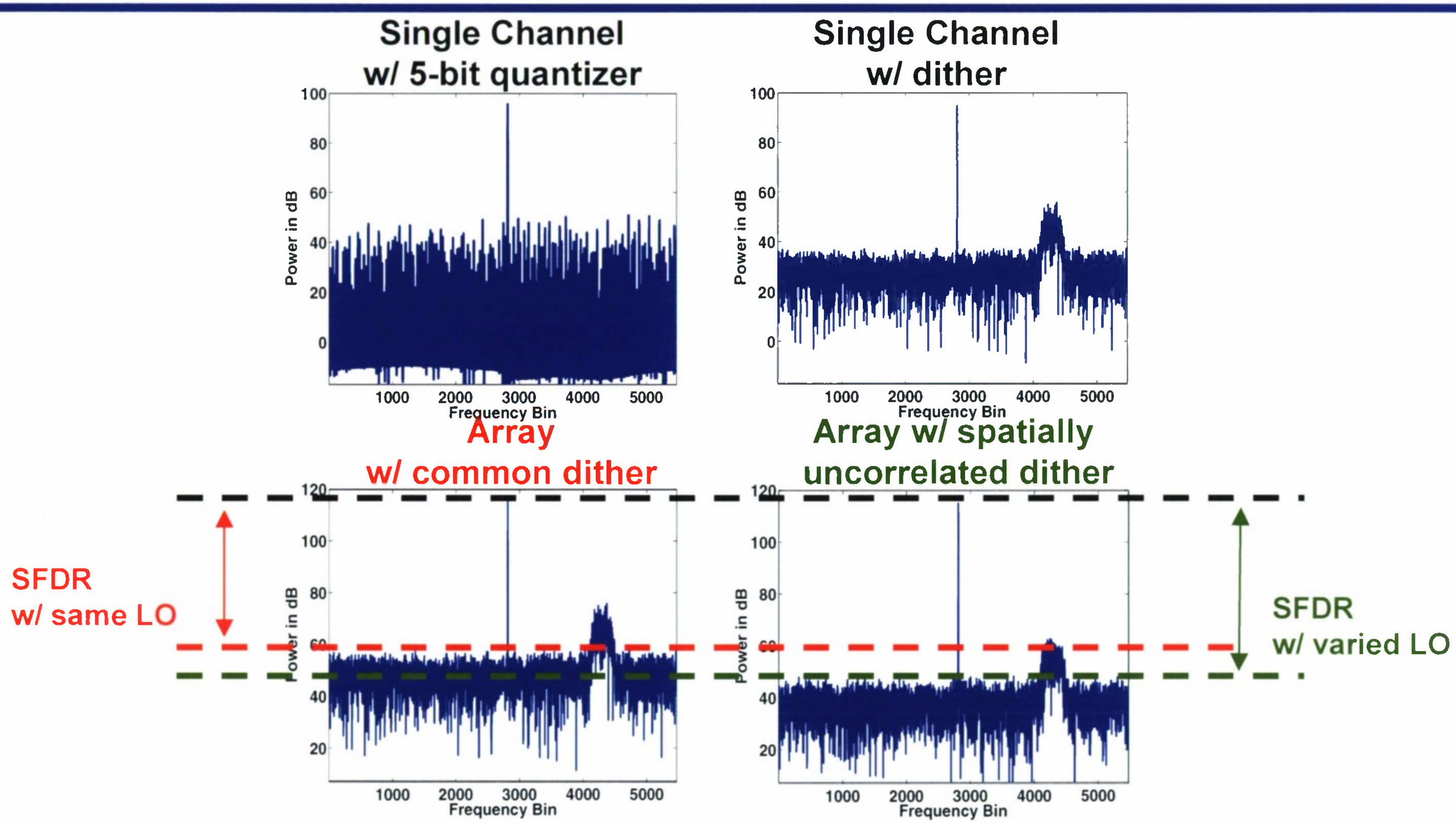
# Predicted DDS Nonlinearities with Receiver-Dependent LO

* 16 bit phase accumulator
6 bit phase word
5 bit DAC



# Example 2: Receiver-Dependent Dither




- Randomized dither makes spurs appear uncorrelated, thus avoiding array gain (ADCs and DACs only)

# Other Types of Distortion

- *M* x *N* products and harmonics can often be addressed by phase transformations

- Intermods from well-separated sources can be addressed by nonlinear-phase all-pass filtering and subsequent equalization

- Closer-in intermods are not addressed directly
  - However, by first mitigating other complex nonlinearities as described (so that they're below the noise), a relatively simple equalizer can be used to remove the remaining close-in intermods (example given later).

- Method above can be applied on transmit in analogous fashion

# Outline

- Introduction
- Importance of linearity and dynamic range
- Method for improving dynamic range
- Experimental data collection
- Future work and summary

# Outline

- Introduction
- Importance of linearity and dynamic range
- Method for improving dynamic range
- Experimental data collection
  - Objectives
  - Mitigation of correlated nonlinearities
  - Uncorrelated nonlinearity
- Future work and summary

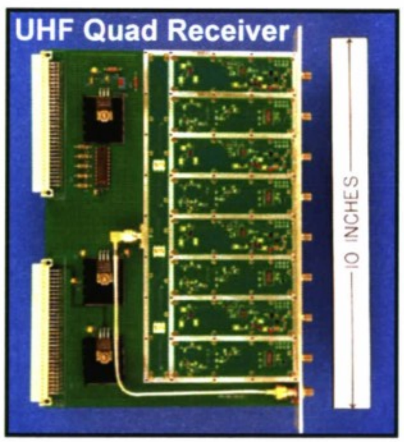
# ADC Testbed Objectives

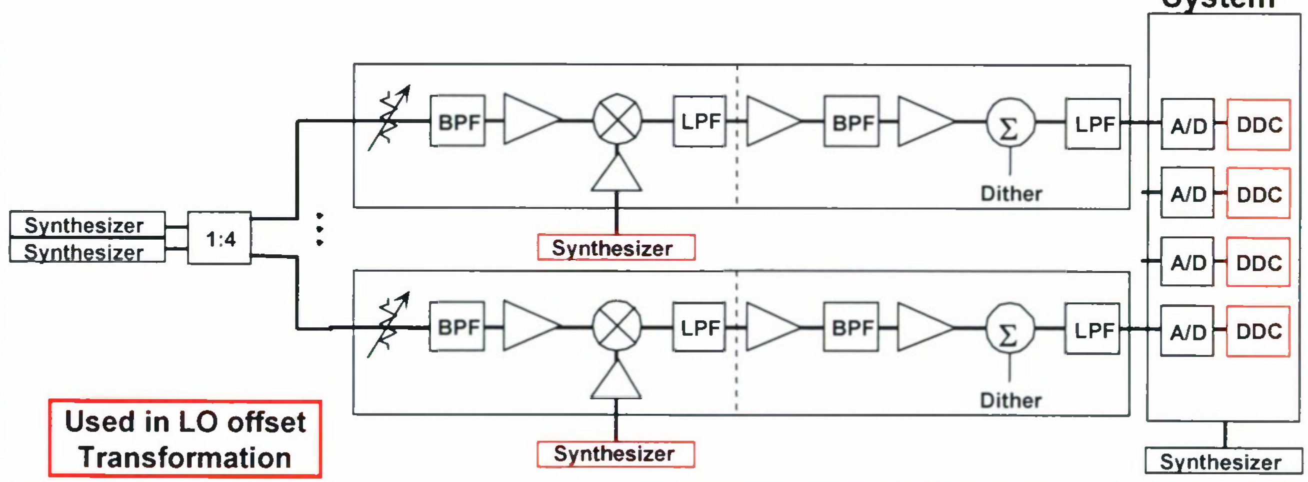
## Goals

- Experiment fit within time and hardware constraints
  - Only four channels available
  - Linear Array Upgrade program had 2-week window of availability for hardware
- Test experimentally whether or not distortions add in beamformer
- Apply mitigation technique to these cases

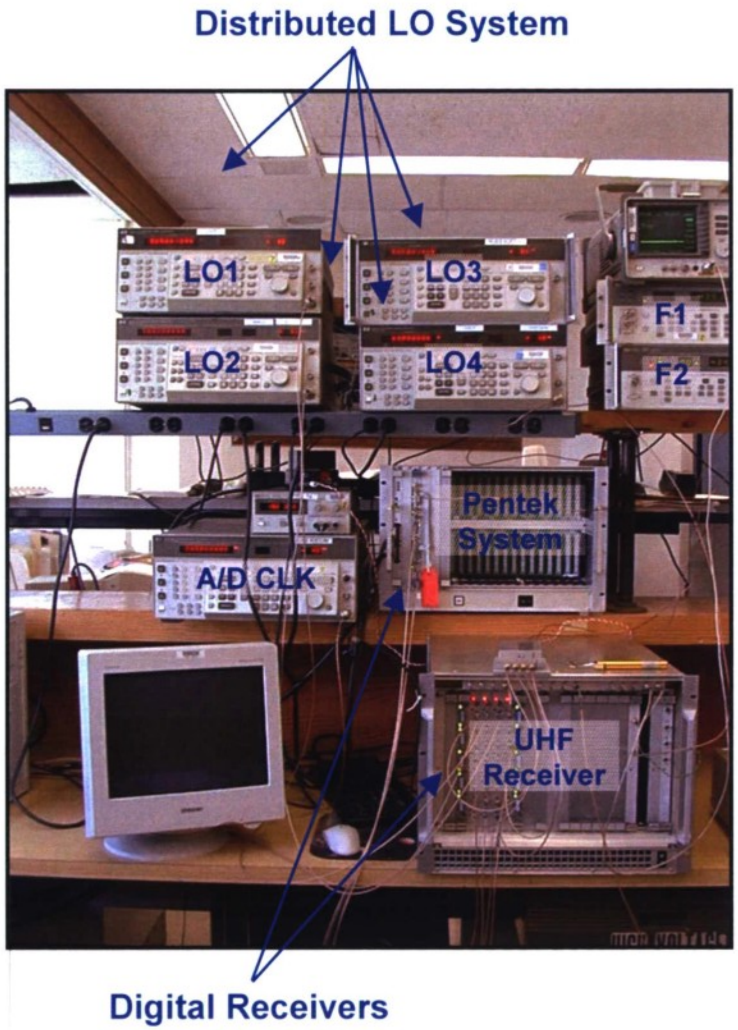
## Hardware

- Group 101 Quad UHF receiver
- Pentek 4290 DSP with pair of 6216 dual digital receiver
  - NCO in DDC has < 1 Hz resolution
  - 12 Bit 65 MSPS ADC
  - Graychip Digital Down-Converter (DDC) GC1016



# Experiments

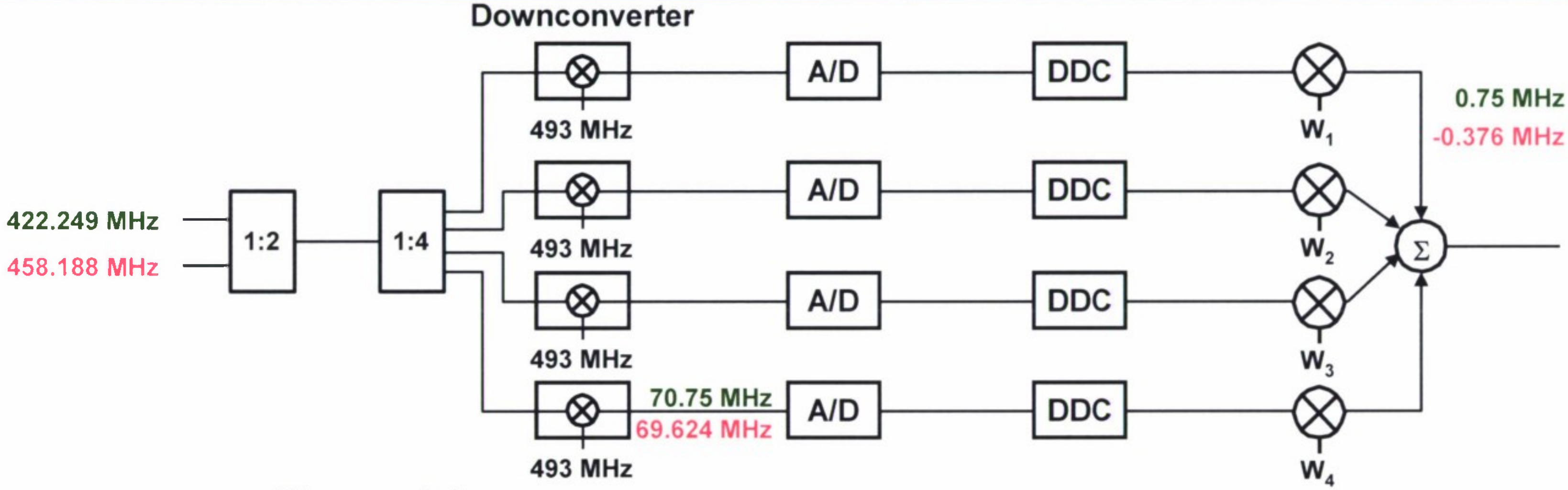


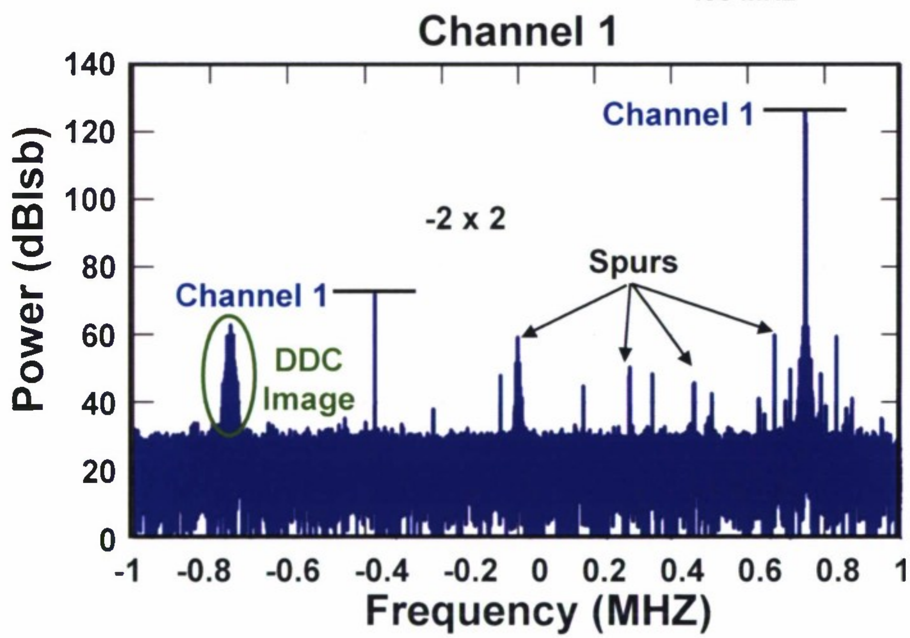

| Case # | | Spur investigated | F1 (MHz) | F2 (MHz) | LO1 (MHz) | LO2 (MHz) | LO3 (MHz) | LO4 (MHz) |
|---|---|---|---|---|---|---|---|---|
| 1 | a | -2 x 2 product | 422 | 458 | 493 | – | – | – |
| | b | -2 x 2 product | 422 | 458 | 493 | $493+\delta_1$ | $493+2*\delta_1$ | $493+3*\delta_1$ |
| | c | -2 x 2 product | 422 | 458 | 493 | $493+\delta_2$ | $493+2*\delta_2$ | $493+3*\delta_2$ |
| 2 | a | -3 x 3 product | 420 | 468 | 491 | – | – | – |
| | b | -3 x 3 product | 420 | 468 | 491 | $491+\delta_1$ | $491+2*\delta_1$ | $491+3*\delta_1$ |
| | c | -3 x 3 product | 420 | 468 | 491 | $491+\delta_2$ | $491+2*\delta_2$ | $491+3*\delta_2$ |
| 3 | a | A/D Harmonic without Dither | 75 | | | | | |
| 4 | a | Receiver and A/D Harmonics | 435 | | 505 | – | – | – |
| | b | Receiver and A/D Harmonics | 435 | | 505 | $505+\delta_1$ | $505+2*\delta_1$ | $505+3*\delta_1$ |
| | c | Receiver and A/D Harmonics | 435 | | 505 | $505+\delta_2$ | $505+2*\delta_2$ | $505+3*\delta_2$ |
| 5 | a | Intermodulation | 435 | 435 | 505 | – | – | – |
| | b | Intermodulation | 435 | 435 | 505 | $505+\delta_1$ | $505+2*\delta_1$ | $505+3*\delta_1$ |
| | c | Intermodulation | 435 | 435 | 505 | $505+\delta_2$ | $505+2*\delta_2$ | $505+3*\delta_2$ |
| | d | Intermodulation | 435 | 435 | 505 | – | – | – |
| | e | Intermodulation | 435 | 435 | 505 | – | – | – |
| 6 | a | Receiver noise | | | 505 | – | – | – |
| | b | Receiver noise | | | 505 | $505+\delta_1$ | $505+2*\delta_1$ | $505+3*\delta_1$ |
| | c | Receiver noise | | | 505 | $505+\delta_2$ | $505+2*\delta_2$ | $505+3*\delta_2$ |
| 7 | a | A/D noise | | | | | | |

# Outline

- Introduction
- Importance of linearity and dynamic range
- Method for improving dynamic range
- Experimental data collection
  - Objectives
  - Mitigation of correlated nonlinearities
  - Uncorrelated nonlinearity
- Future work and summary

# -2 x 2 Mixer Intermodulation Product Correlation

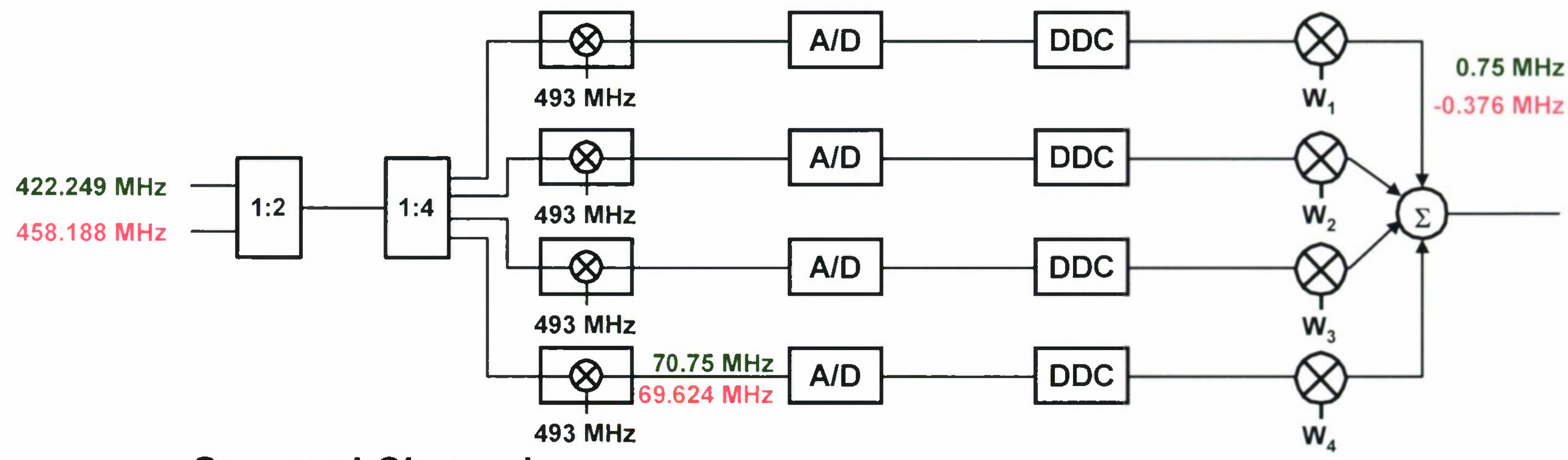


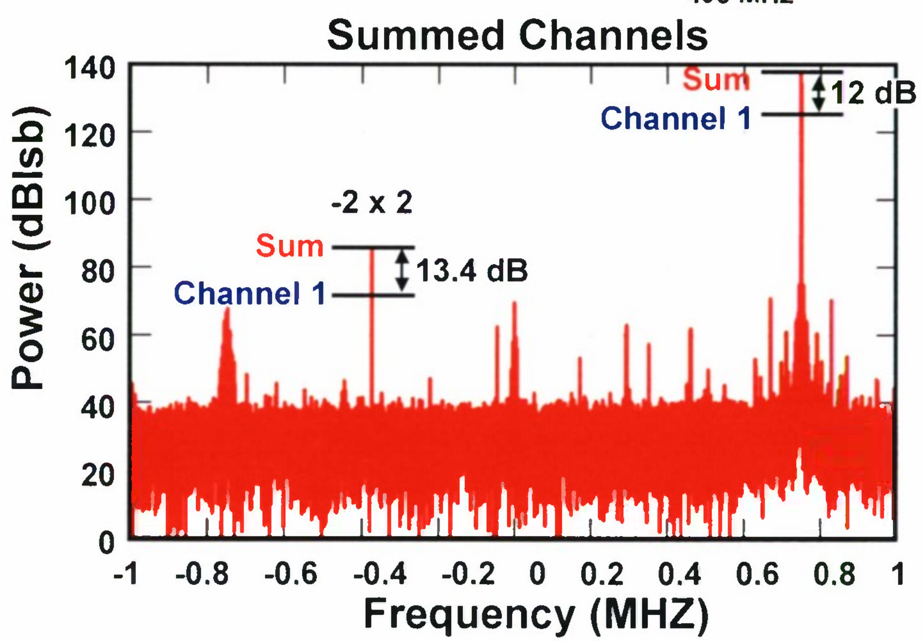




**Legend**

Channel 1

# -2 x 2 Mixer Intermodulation Product Correlation

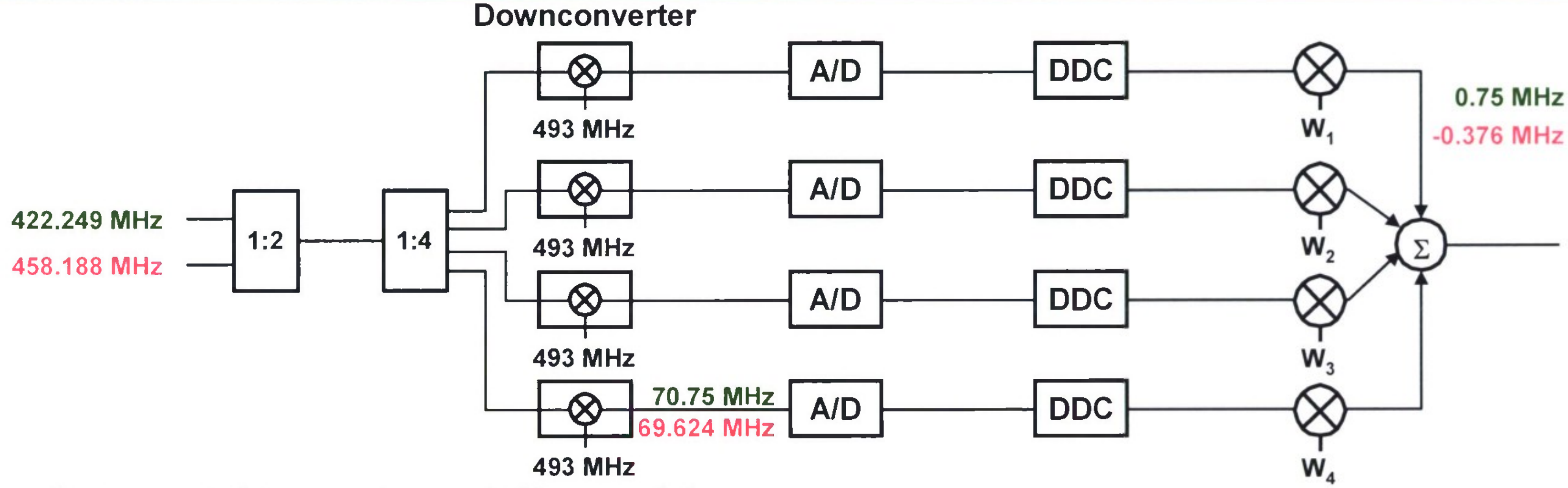


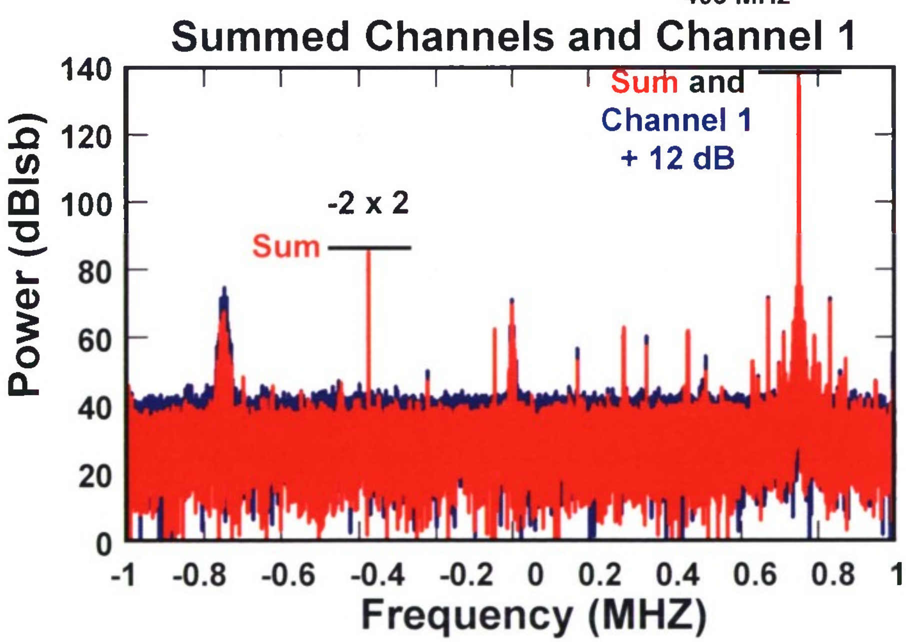




Legend

Sum

# -2 x 2 Mixer Intermodulation Product Correlation



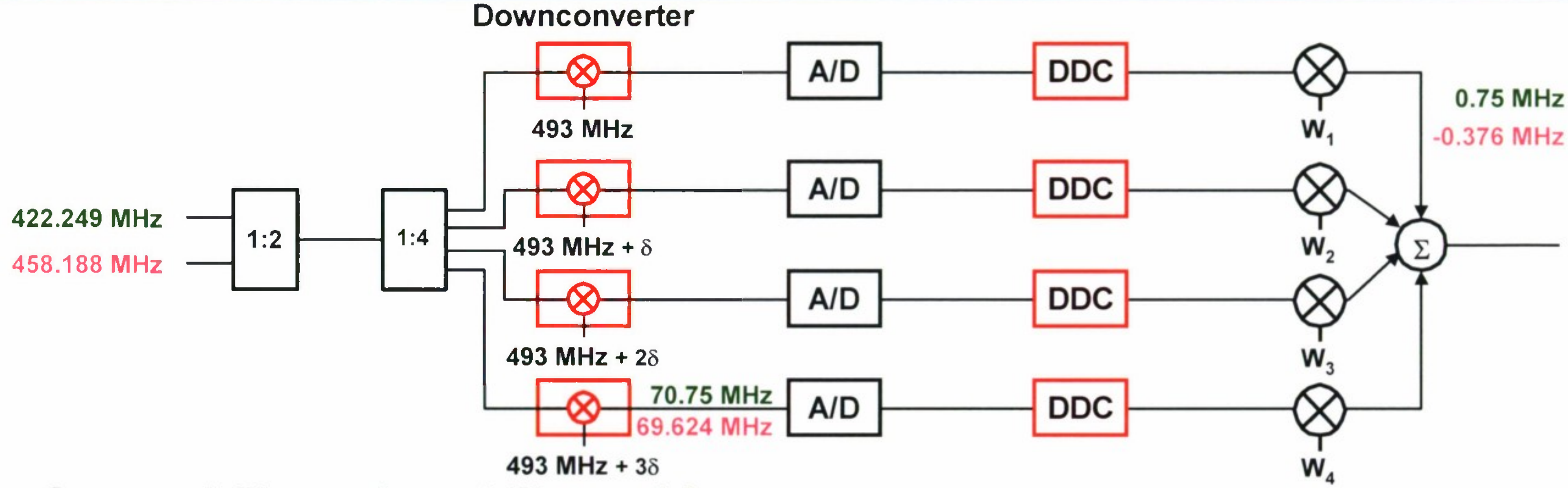

**Legend**

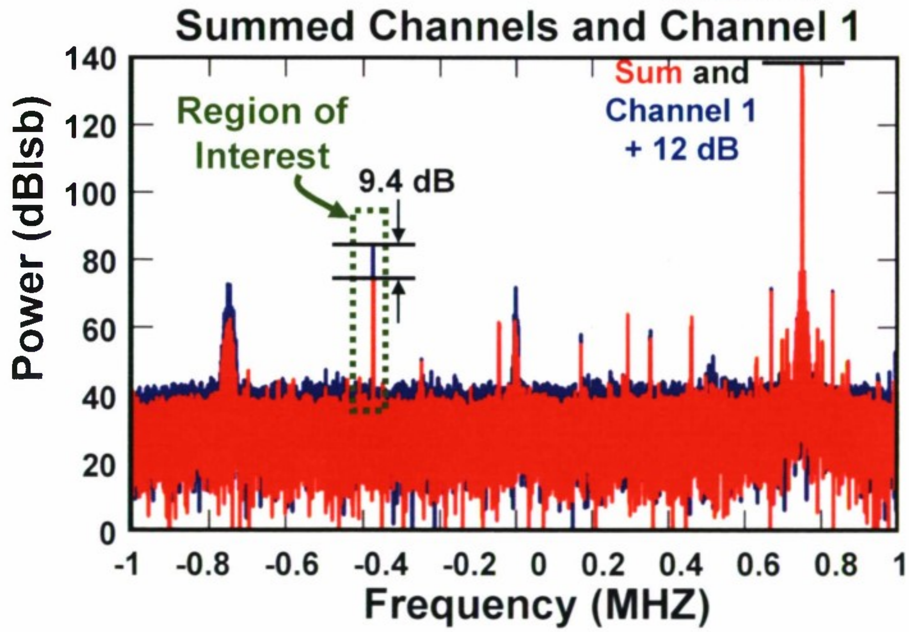
Channel 1 + 12 dB

Sum

-2 x 2 Spur is Correlated Across Channels, and Limits Dynamic Range Gain

# LO Offset Mitigation Applied to -2 x 2 Mixer Intermodulation Product Correlation




**Legend**

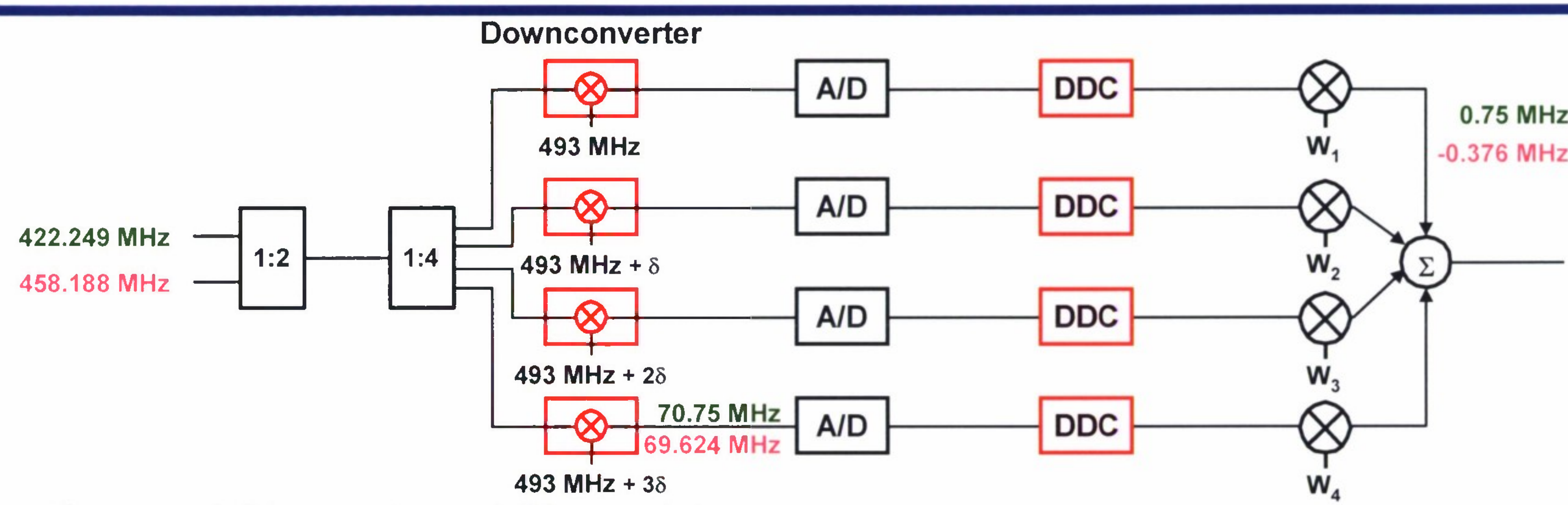
Channel 1 + 12 dB

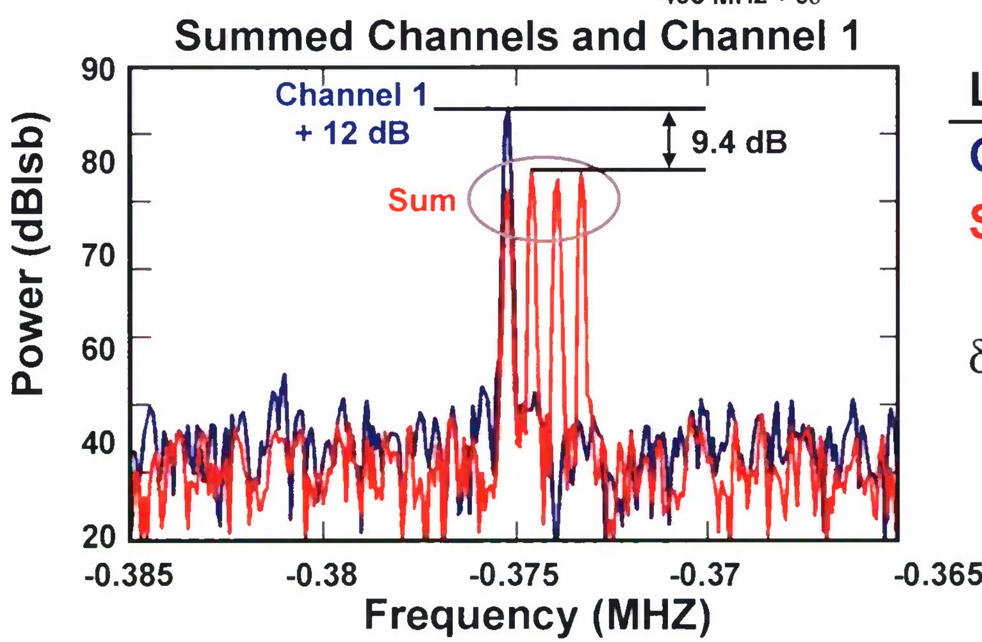
Sum

$\delta$ = 684 Hz

LO Offset Mitigation Technique Decorrelates -2 x 2 Spur

# LO Offset Mitigation Applied to -2 x 2 Mixer Intermodulation Product Correlation




Legend

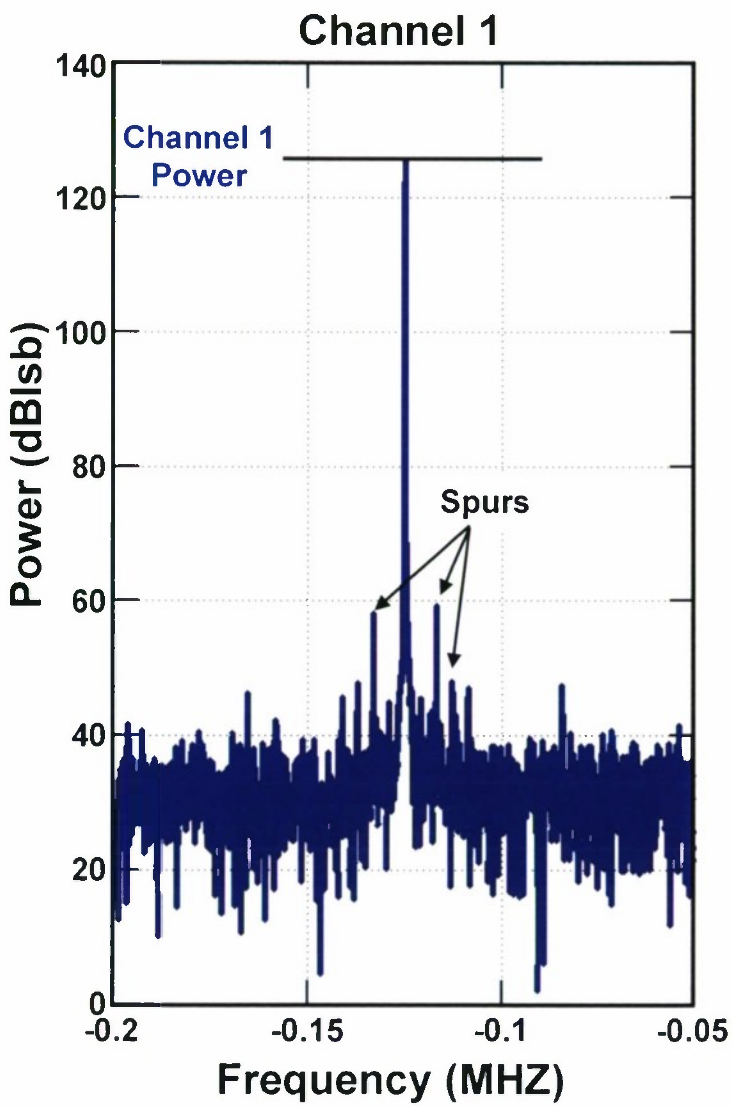
Channel 1 + 12 dB

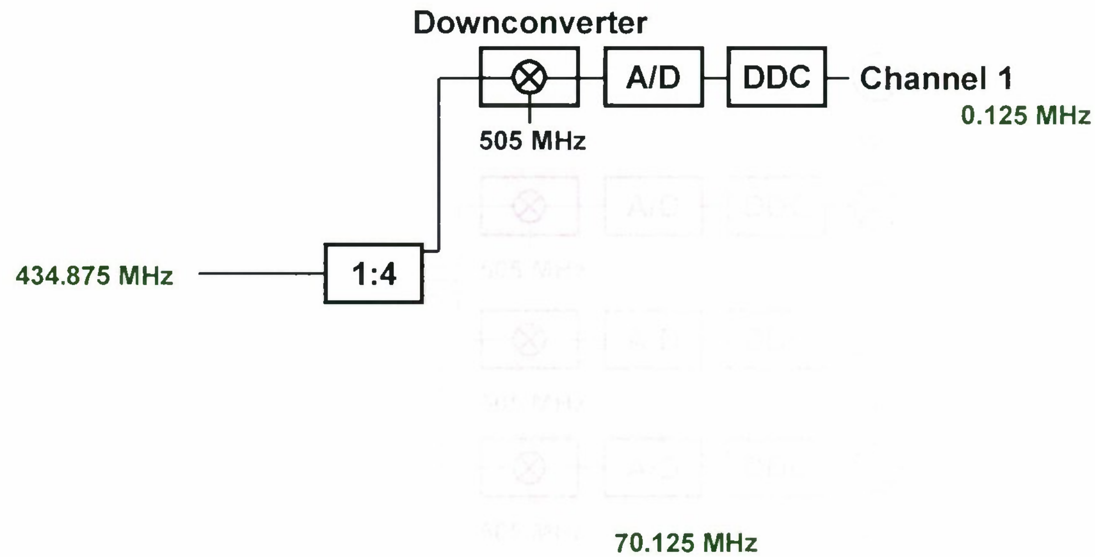
Sum

$\delta$ = 684 Hz

LO Offset Moves Spur in Each Channel to a Different Frequency

# Noise Correlation

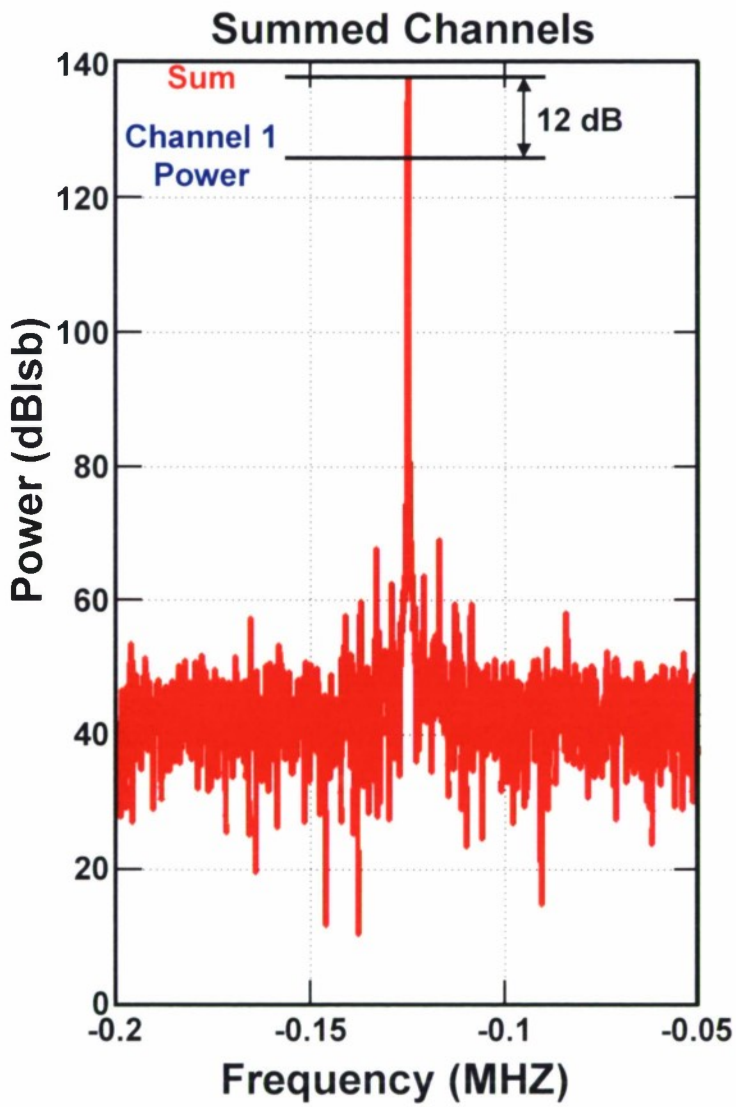


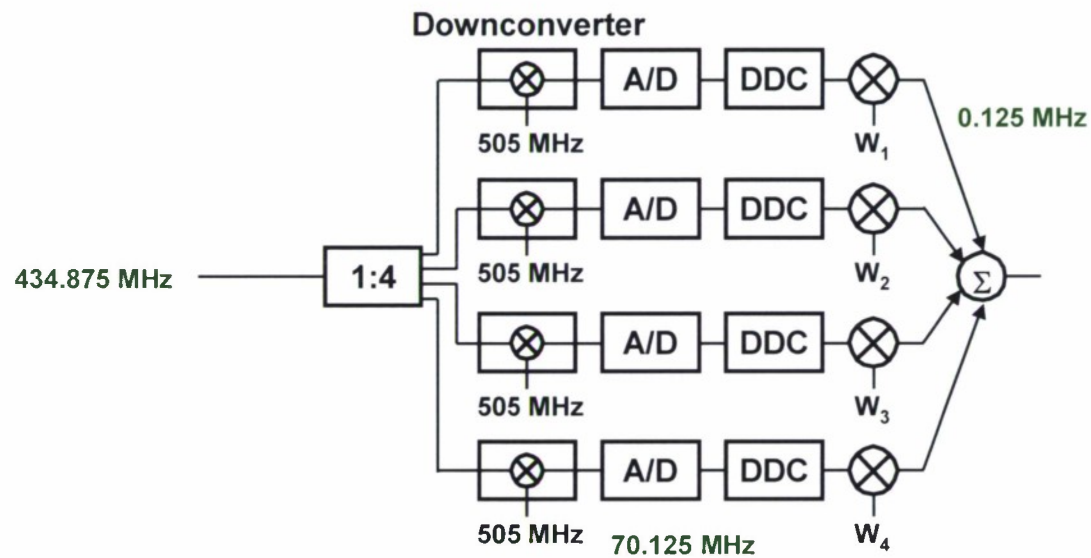




**Legend**

**Channel 1**

# Noise Correlation

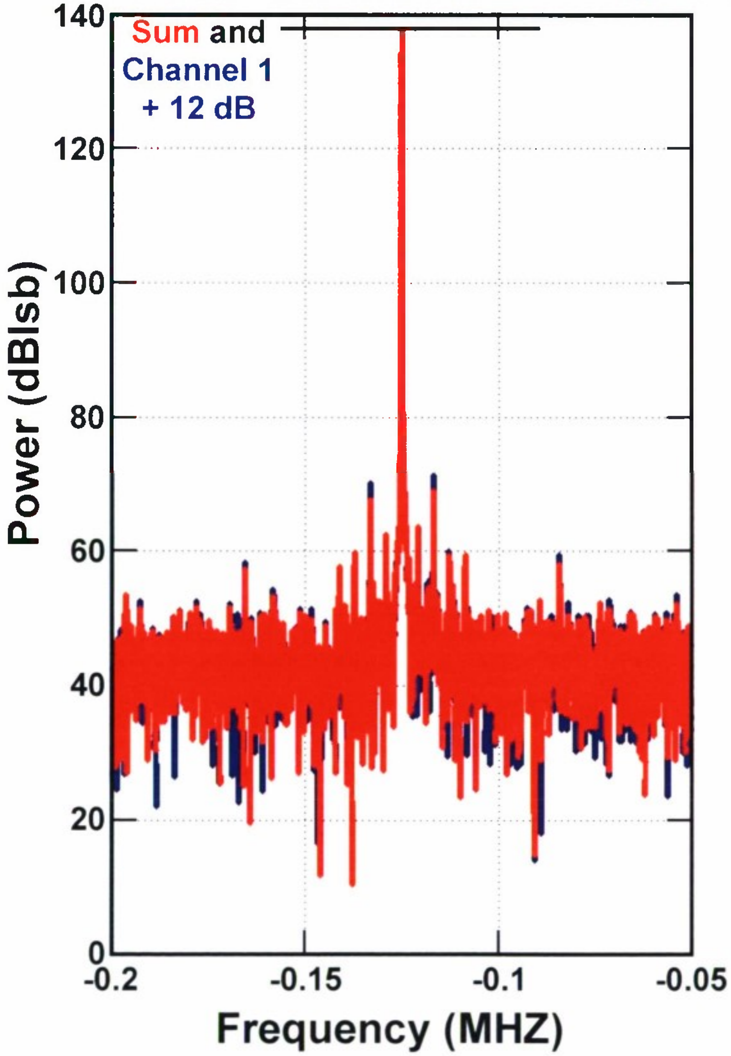


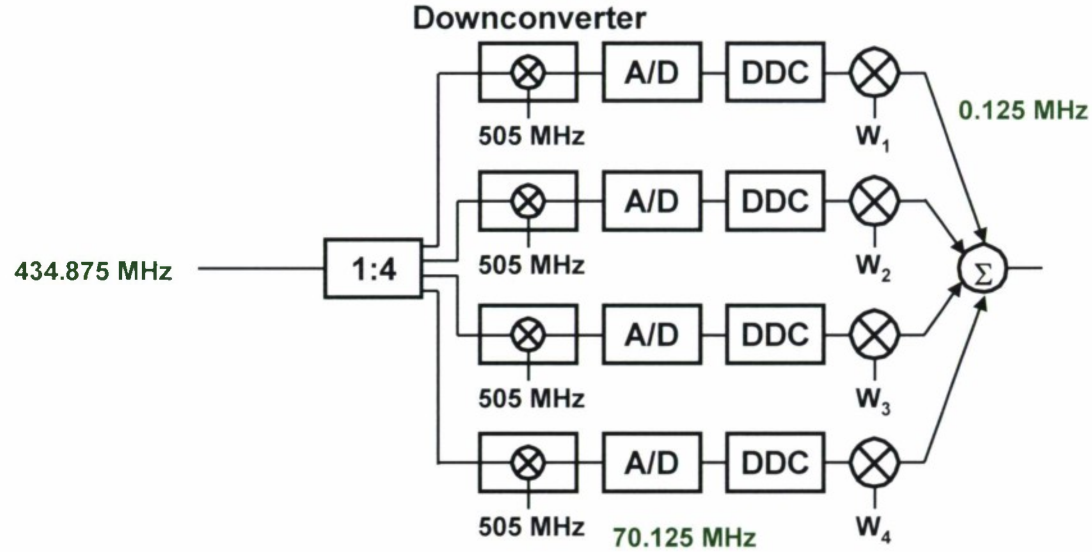




## Legend

Sum of 4 Channels

# Noise Correlation




**Legend**

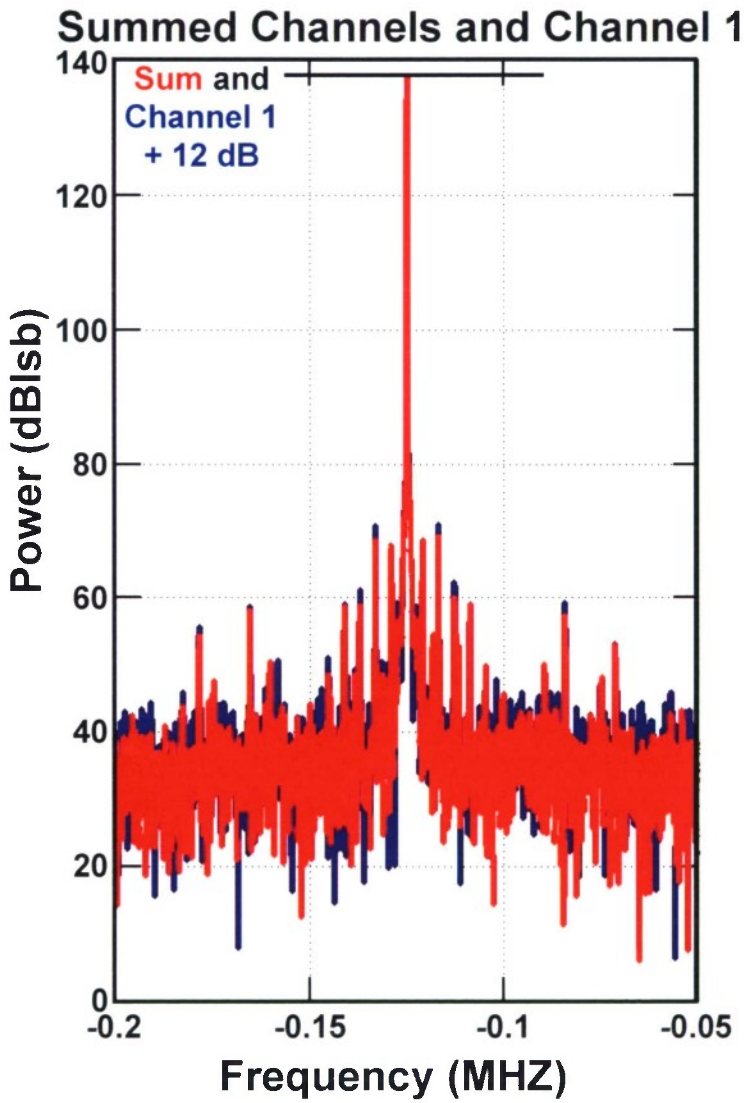
**Channel 1 + 12 dB**

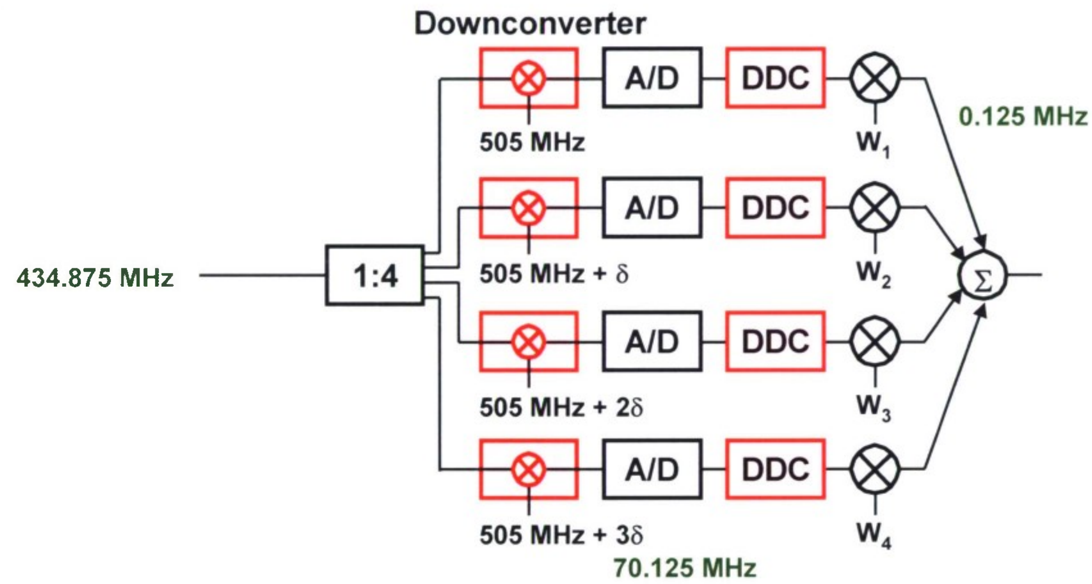
**Sum of 4 Channels**

SNR of 4 Channels only 0.8 dB higher (vs 6 dB expected)

$\Rightarrow$Phase Noise is correlated!

# LO Offset Mitigation Applied to Noise Correlation Data



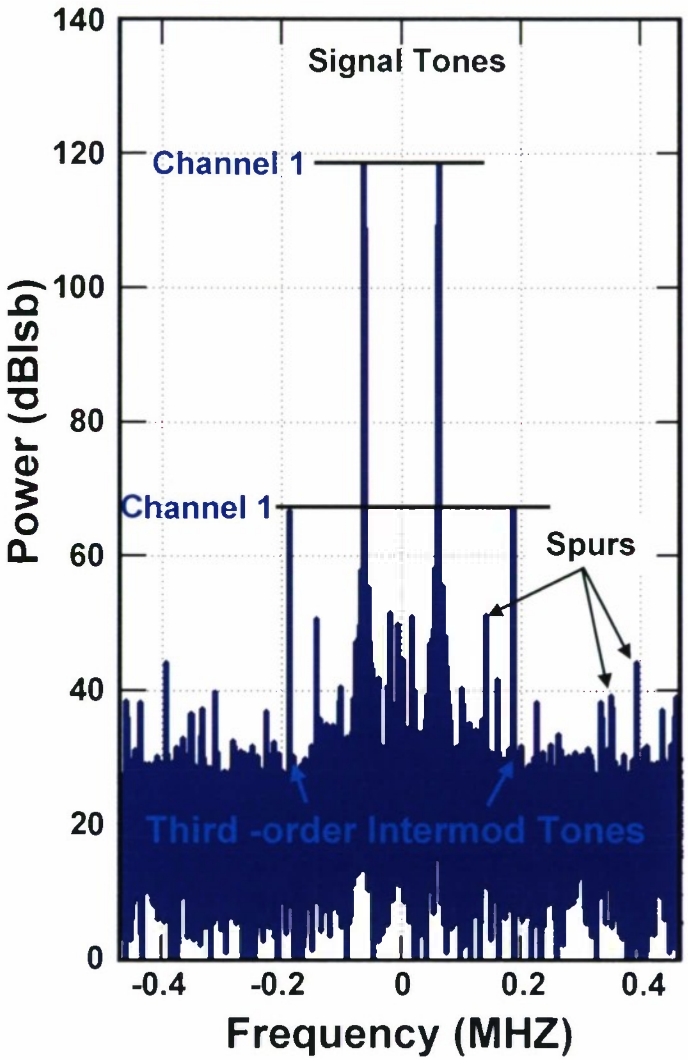

**Legend**

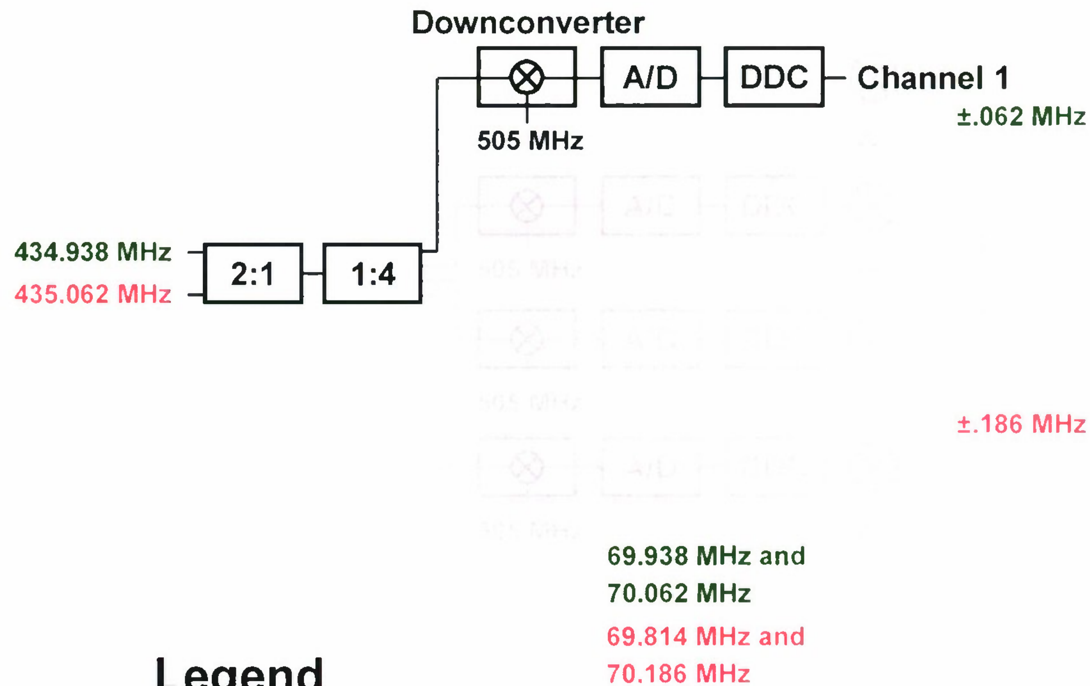
Channel 1 + 12 dB

Sum of 4 Channels

SNR Gain on Beamforming is Now 4.1 dB (vs 0.8 dB)

# Third-order Intermodulation Correlation

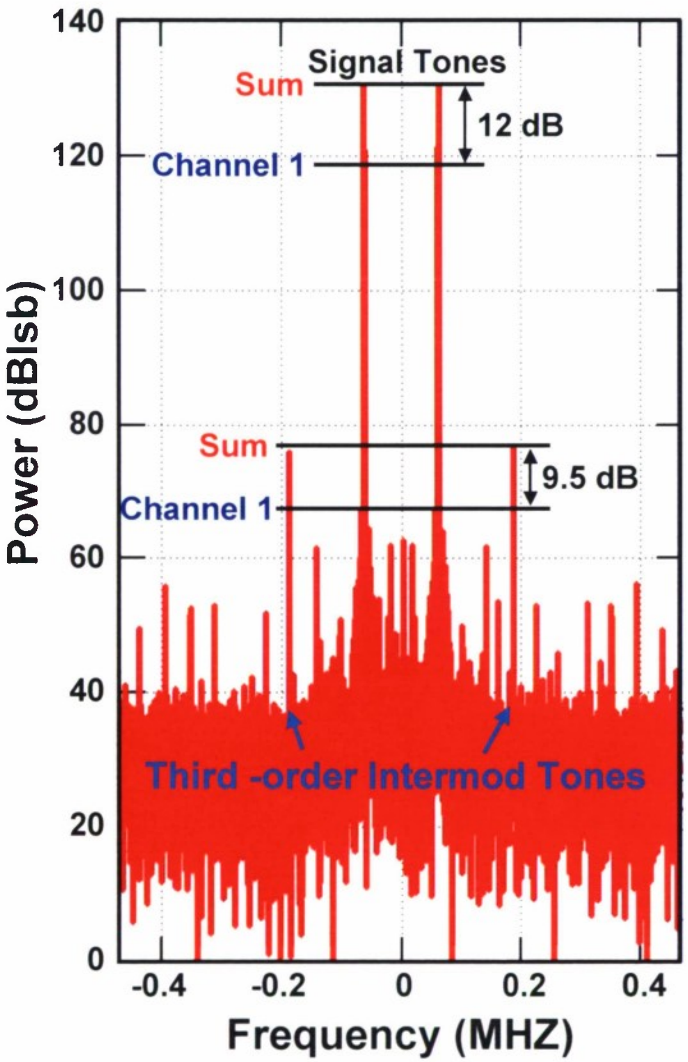


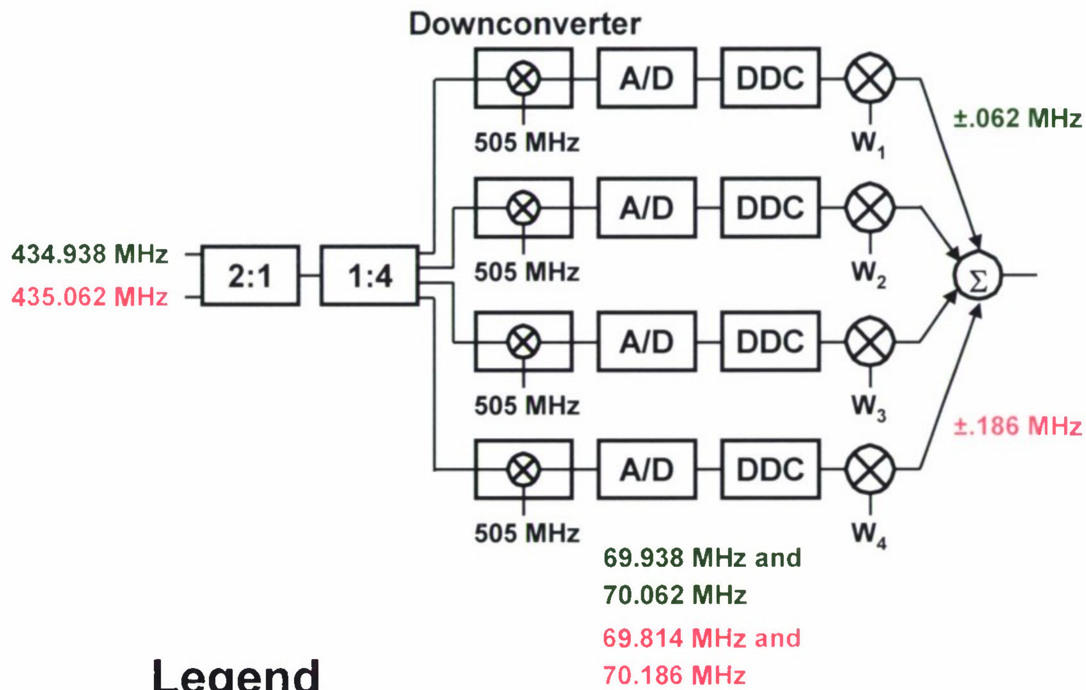




Legend

Channel 1

# Third-order Intermodulation Correlation



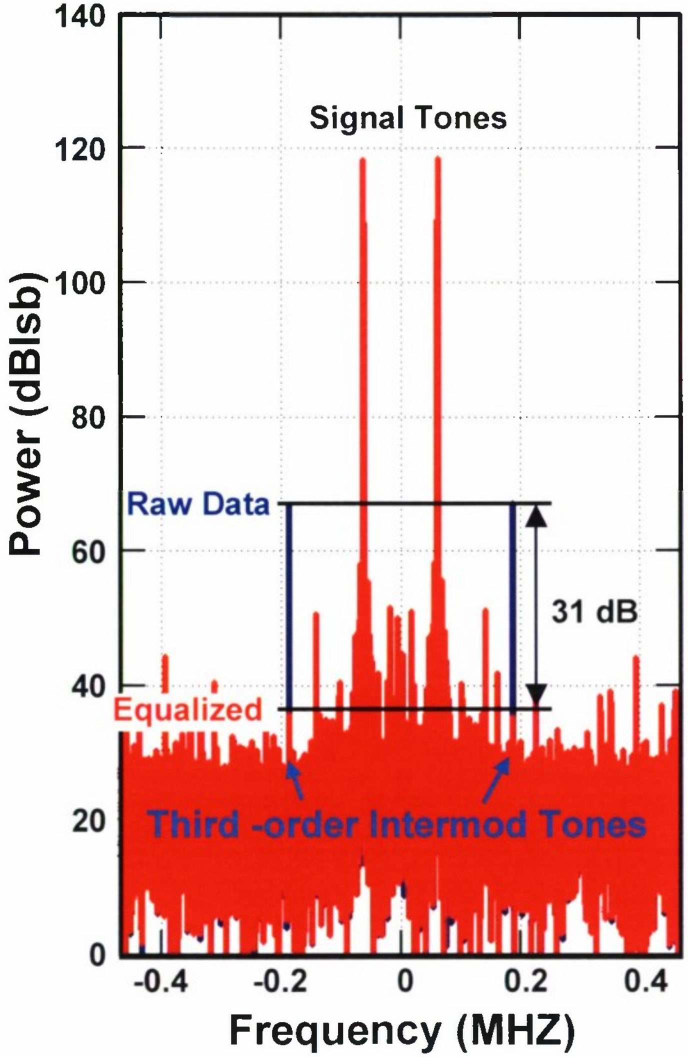




## Legend

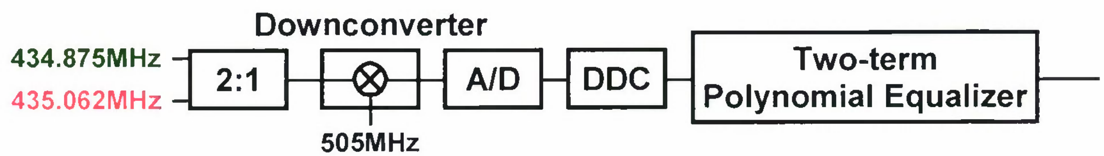
Sum of 4 Channels

Third-order intermodulation distortion is correlated

# Third-order Intermodulation Mitigation




*Approach: Mitigate most undesired terms by decorrelating them, as described above. Then use a simple, low-order nonlinear equalizer to cancel the rest.*




**Legend**

**Raw Data**

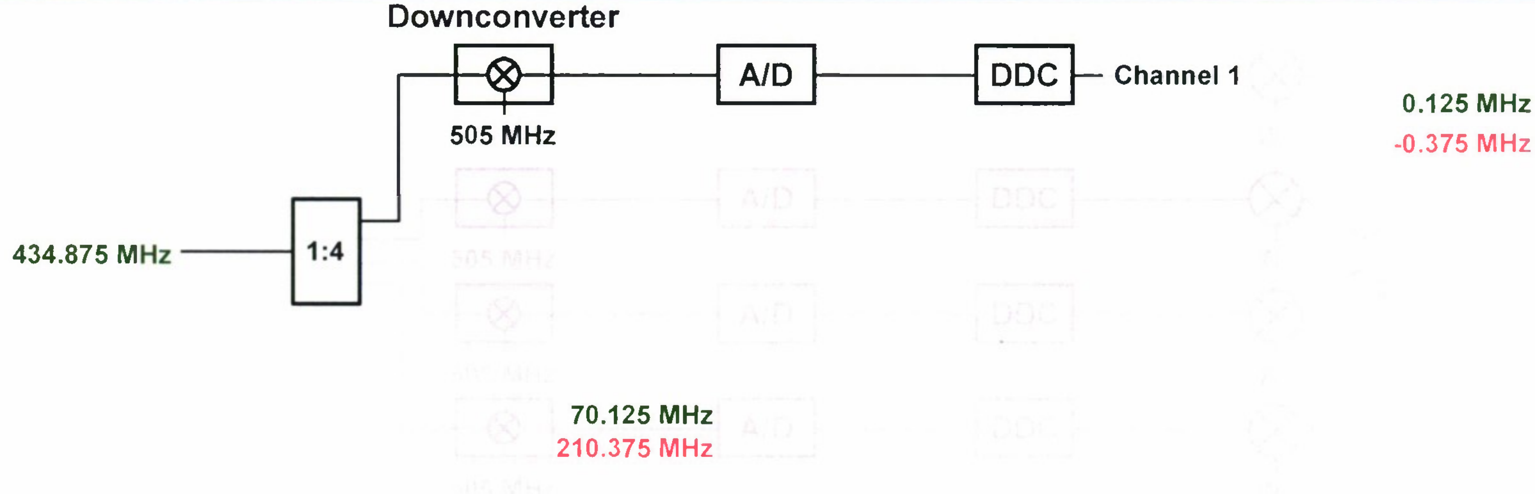
**Equalized**

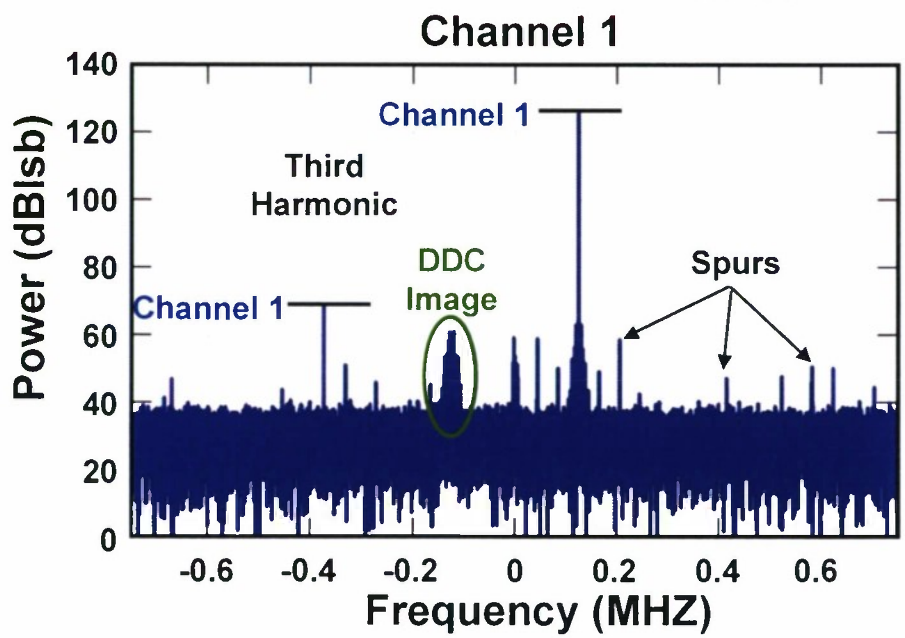
Nonlinear Equalization Reduces Third-order Intermods by 31 dB

# Outline

- Introduction
- Importance of linearity and dynamic range
- Method for improving dynamic range
- Experimental data collection
  - Objectives
  - Mitigation of correlated nonlinearities
  - Uncorrelated nonlinearity
- Future work and summary

# Third Harmonic Correlation

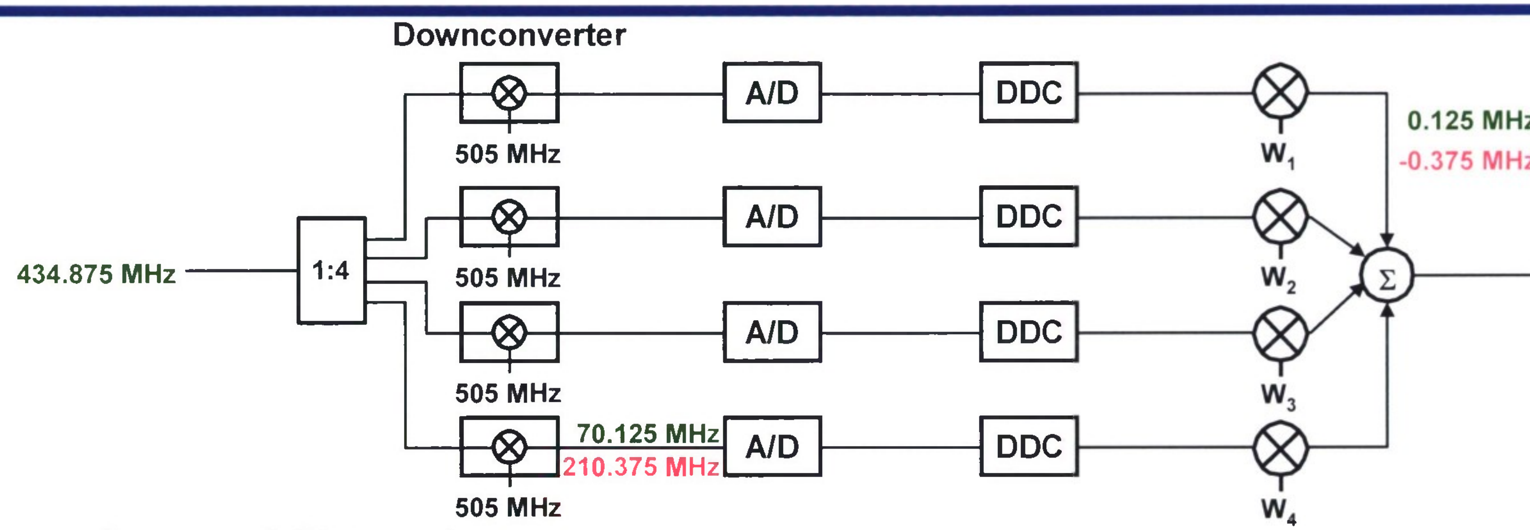


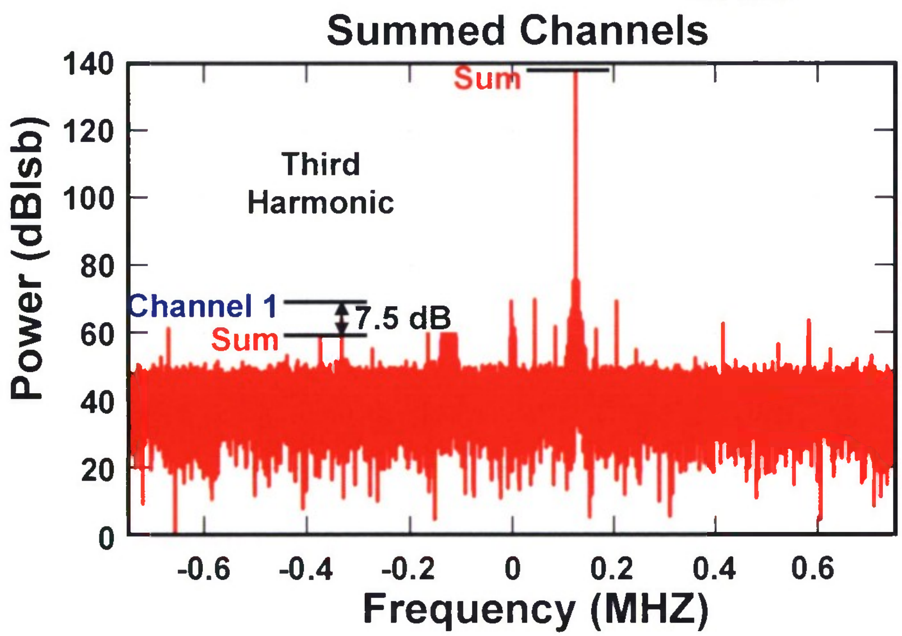




Legend

Channel 1

# Third Harmonic Correlation

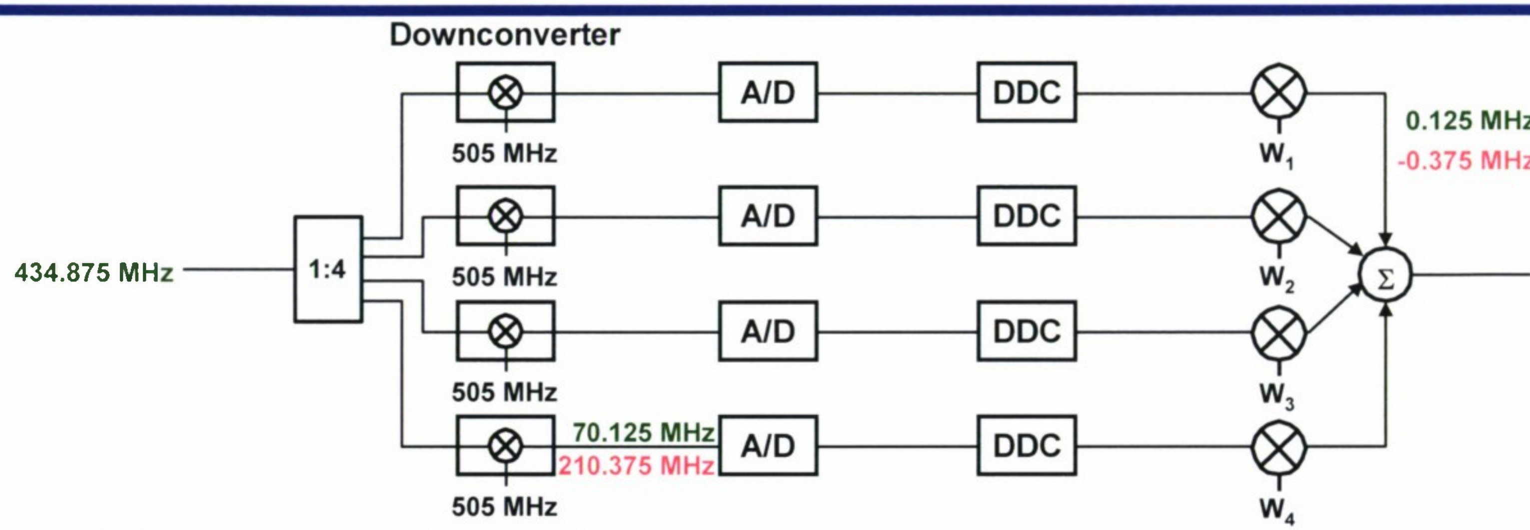


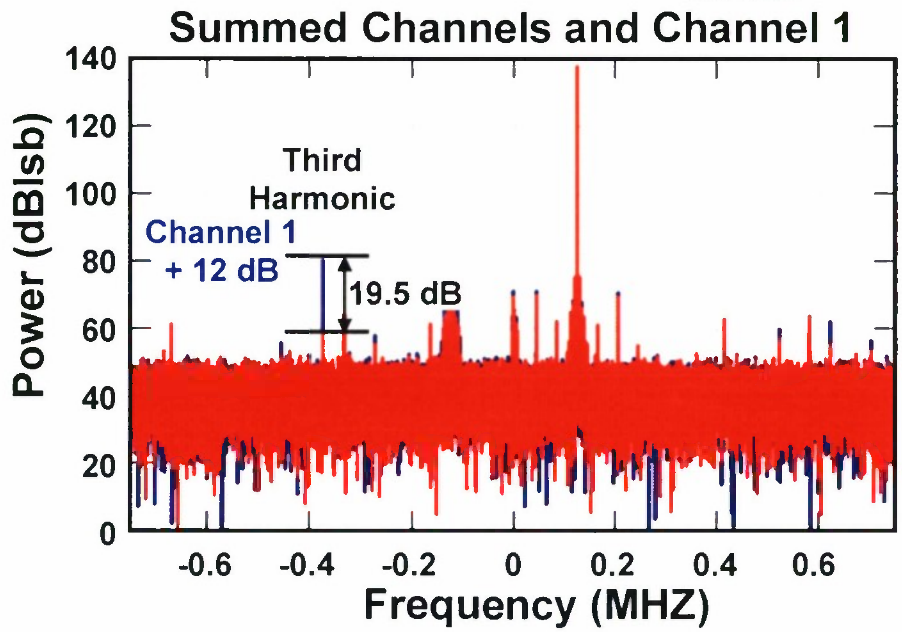




## Legend

Sum

# Third Harmonic Correlation




**Legend**

Channel 1

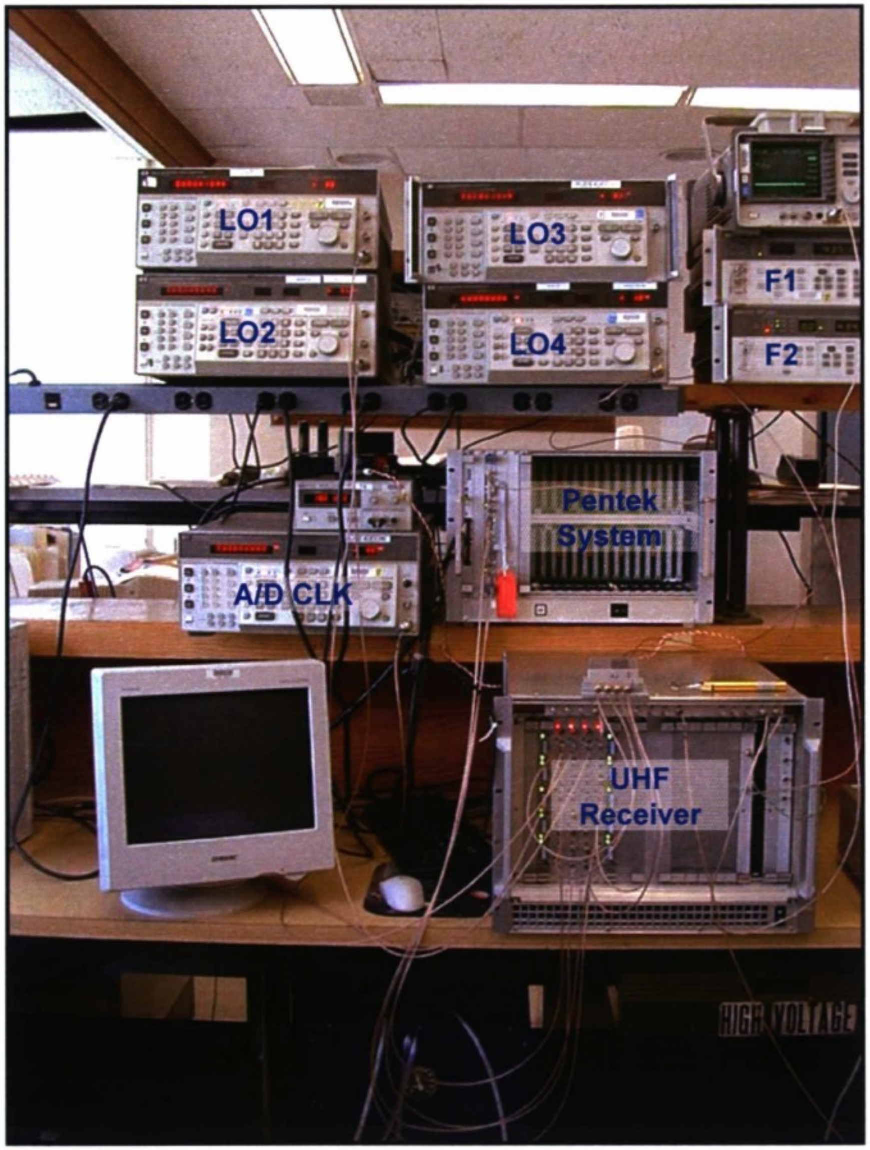
Sum

**Third Harmonic is Uncorrelated**

# Outline

- Introduction
- Importance of linearity and dynamic range
- Method for improving dynamic range
- Experimental data collection
- Future work and summary

# Future Work




- Shrink synthesizers
  - DDS
  - Phase-locked sources
- Other transformations
- Investigate more channels when Linear Array Upgrade hardware is available

# Summary

- Many sources of nonlinearities in a digital TR module
- Correlation of nonlinearities will reduce system dynamic range
  - Measurements show that some distortions are strongly correlated, while others are not
- Proposed method "Digital Array Nonlinearity Reduction" to make nonlinearities appear uncorrelated and/or incoherent from channel to channel
  - Several manifestations described
  - Simulations used to examine the effect on some specific nonlinearities
  - Technique has been verified experimentally

# REPORT DOCUMENTATION PAGE

*Form Approved*
*OMB No. 0704-0188*



**1. REPORT DATE** *(DD-MM-YYYY)*
1 November 2004

**2. REPORT TYPE**

**3. DATES COVERED** *(From - To)*

**4. TITLE AND SUBTITLE**
Correlation of Nonlinear Distortion in Digital Phased Arrays: Measurement and Mitigation

**5a. CONTRACT NUMBER**
F19628-00-C-0002

**5b. GRANT NUMBER**

**5c. PROGRAM ELEMENT NUMBER**

**6. AUTHOR(S)**
D. J. Rabideau and L. C. Howard

**5d. PROJECT NUMBER**

**5e. TASK NUMBER**

**5f. WORK UNIT NUMBER**

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
MIT Lincoln Laboratory
244 Wood Street
Lexington, MA 02420-9108

**8. PERFORMING ORGANIZATION REPORT NUMBER**
101-1037

**9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
Lincoln Laboratory Advanced Concepts Program
244 Wood Street
Lexington, MA 02430-9108

**10. SPONSOR/MONITOR'S ACRONYM(S)**

**11. SPONSOR/MONITOR'S REPORT NUMBER(S)**
ESC-TR-2004-086

**12. DISTRIBUTION / AVAILABILITY STATEMENT**

Approved for public release; distribution is unlimited.

**13. SUPPLEMENTARY NOTES**

**14. ABSTRACT**

In a digital array, each receiver performs analog-to-digital-conversion (ADC), with the resulting digital data later combined via digital beamforming techniques. Since ADC is performed prior to beamforming, it is thus theoretically possible to enhance the dynamic-range (DR) of the individual receivers through post-ADC array integration gain. In practice, however, DR enhancement is limited by correlation of the nonlinearities (from receiver to receiver). Worse still, little published data exists on this subject (i.e., suitable for quantitatively assessing the correlation coefficients). This makes it difficult to predict how much DR enhancement will be achieved on real digital arrays.

This presentation describes the results of recent experiments involving a four-channel digital receiver system. The system was used to measure the correlation (between receivers) of different types of nonlinear distortion. The measurements quantitatively demonstrate that some nonlinearities are highly correlated. Next, the system was used to evaluate a recently proposed method for decorrelating nonlinear distortion in digital arrays. The measurements show that the mitigation technique is successful in decorrelating some nonlinear signal components.

**15. SUBJECT TERMS**

| | | | |
|---|---|---|---|
| arrays | radar | phase noise | intermodulation |
| digital arrays | dynamic range | spurs | digital receivers |

| 16. SECURITY CLASSIFICATION OF: a. REPORT | b. ABSTRACT | c. THIS PAGE | 17. LIMITATION OF ABSTRACT | 18. NUMBER OF PAGES | 19a. NAME OF RESPONSIBLE PERSON / 19b. TELEPHONE NUMBER *(include area code)* |
|---|---|---|---|---|---|
| Unclassified | Unclassified | Unclassified | Same as report | 43 | |



**Standard Form 298 (Rev. 8-98)**
**Prescribed by ANSI Std. Z39.18**